STAT/MA 41600
In-Class Problem Set #1: August 27, 2014

**1.** Consider a sample space $S$ with seven outcomes, e.g., $S = \{a, b, c, d, e, f, g\}$.
(a.) How many events are there, which contain outcome $a$?
(b.) How many events are there, which contain outcome $a$ and outcome $b$?
(c.) How many events are there which have exactly five outcomes?
(d.) How many events are there which have four or more outcomes?
(e.) How many events are there which have at most two (i.e., two or fewer) outcomes?

**2a.** A point is chosen at random inside in the quadrilateral in Figure 1(a). What is the sample space? [Hint: It might be helpful to break the quadrilateral into two or more pieces.]
**2b.** A point is chosen at random in the triangle in Figure 1(b). What is the sample space? [Hint: It might be helpful to give bounds on $x$ and then give bounds on $y$.]
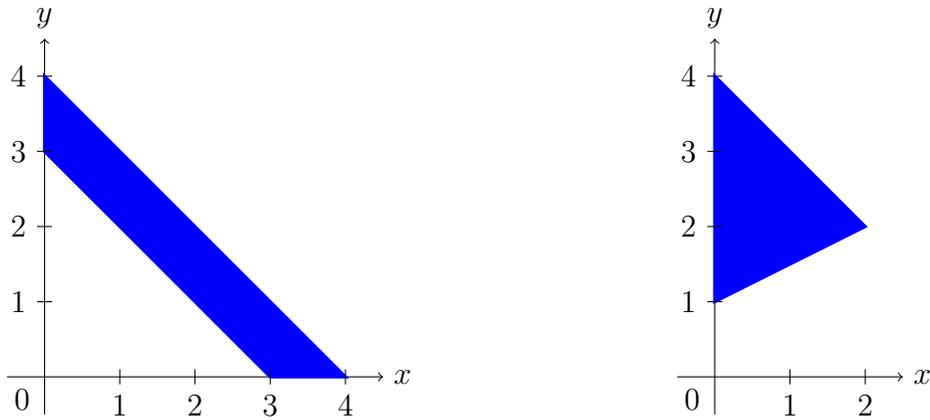


Figure 1: (a.) A quadrilateral. (b.) A triangle.

**3.** Alice, Bob, Catherine, Doug, and Edna are randomly assigned seats at a circular table in a perfectly circular room. Assume that rotations of the table do not matter, so there are exactly 24 possible outcomes in the sample space. Alice, Catherine, and Edna are sisters. Sisters love to sit together. In how many of these 24 outcomes are all three sisters sitting together (in an adjacent cluster) and are therefore happy?

**4.** Consider a person who starts at the origin in the Cartesian plane (i.e., in the $(x, y)$ plane). He takes four steps, each of length 1. Each step is left, right, up, or down. So there are $4^4 = 256$ outcomes, corresponding to the 256 paths that he might choose to take. How many outcomes (i.e., 4-step-paths) will put him back at the origin, after the four steps are done?

**5.** Six rocks are sitting in a straight line. We paint them, using up to three colors (say, $R$'s, $W$'s, and $B$'s). If the colors of the rocks are listed, left-to-right, then one possible outcome is $(R, B, B, B, W, B)$. Another possible outcome is $(B, W, W, W, B, W)$. Etc... The sample space—consisting of all possible outcomes of the painting—has $3^6 = 729$ possible outcomes. How many of the 729 outcomes have the property that each color is used exactly two times?

**6.** Simplify the expression $e^{-1} \sum_{j=3}^{\infty} \frac{1}{j!}$.

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