

Problem Set 1 Answers

1a. When building an event that contains outcome a , we have 6 choices to make, each with two possibilities: Should b be included, or not? Should c be included, or not? Should d be included, or not? Should e be included, or not? Should f be included, or not? Should g be included, or not? So there are $2^6 = 64$ ways to build an event containing a .

1b. When building an event containing a and b , there are 5 choices to make (similar to the answer above), each with two possibilities. So there are $2^5 = 32$ events containing a and b .

1c. We choose 5 out of 7 outcomes to make the event, so there are $\binom{7}{5} = \frac{7!}{5!2!} = 21$ such events.

1d. There are $\binom{7}{4} = 35$ events with 4 outcomes; $\binom{7}{5} = 21$ events with 5 outcomes; $\binom{7}{6} = 7$ events with 6 outcomes; $\binom{7}{7} = 1$ event with 7 outcomes. So there are $35 + 21 + 7 + 1 = 64$ events with 4 or more outcomes. [Another method is to use symmetry: There are the same number of events containing 7, 6, 5, or 4 outcomes, as there are events containing 0, 1, 2, or 3 outcomes. So half of the 128 events (namely, 64 events) have 4 or more outcomes.]

1e. There are $\binom{7}{2} = 21$ events with 2 outcomes; $\binom{7}{1} = 7$ events with 1 outcome; $\binom{7}{0} = 1$ event with 0 outcomes. So there are $21 + 7 + 1 = 29$ events with 2 or fewer outcomes.

2a. We split the sample space to the left or right of the line $x = 3$. So the sample space is $S = \{(x, y) \mid 0 \leq x \leq 3, 3 - x \leq y \leq 4 - x\} \cup \{(x, y) \mid 3 \leq x \leq 4, 0 \leq y \leq 4 - x\}$.

2b. The sample space is $S = \{(x, y) \mid 0 \leq x \leq 2, \frac{1}{2}x + 1 \leq y \leq 4 - x\}$.

3. The sisters sit together if and only if Bob and Doug are next to each other. Regardless of where Bob sits, there are four seats possible for Doug, two of which are next to Bob. So Bob and Doug are next to each other in $2/4 = 1/2$ of the outcomes, i.e., in 12 of the outcomes. [Alternative explanation: If the girls sit together, they have $3! = 6$ orderings, and for each such way, there are 2 orderings for the boys. So there are $(6)(2) = 12$ such outcomes.]

4. There are $4! = 24$ paths that have 1 step of each type. There are $\binom{4}{2} = 6$ paths consisting of two up's and two down's. There are $\binom{4}{2} = 6$ paths consisting of two left's and two right's. So there are $24 + 6 + 6 = 36$ such paths altogether.

5. There are $\binom{6}{2} = 15$ ways to paint two of the six rocks blue. For each such way, there remain $\binom{4}{2} = 6$ ways to paint two of the four remaining rocks red. Finally, the last two rocks must be white. So there are $(15)(6) = 90$ such outcomes (with two of each color) altogether.

6. We know $e^x = \sum_{j=0}^{\infty} \frac{x^j}{j!}$. So $e^1 = \sum_{j=0}^{\infty} \frac{1}{j!}$. Subtracting the $j = 0, 1, 2$ terms from both sides, we obtain $e - 1 - 1 - 1/2 = \sum_{j=3}^{\infty} \frac{1}{j!}$. Multiplying by e^{-1} on both sides, we have $e^{-1}(e - 5/2) = e^{-1} \sum_{j=3}^{\infty} \frac{1}{j!}$. Equivalently, $1 - (5/2)e^{-1} = e^{-1} \sum_{j=3}^{\infty} \frac{1}{j!}$.