

STAT/MA 41600  
In-Class Problem Set #2: August 29, 2014

1. Roll three (6-sided) dice.
  - 1a. Use inclusion-exclusion to find the probability that *at least one* value of “2” appears.
  - 1b. Find the probability that *at most one* value of “2” appears.
2. Is it always true that, if  $A$  and  $B$  are events (maybe overlapping, maybe disjoint), then  $P(A \cup B) \leq P(A) + P(B)$ ? If “yes,” then give a short and simple reason why this is always true. If “no,” then give a counterexample, i.e., a situation in which  $P(A \cup B) > P(A) + P(B)$ .
- 3a. Two red and two blue plates are placed at a circular table for four people, with all arrangements of the plates equally likely. Find the probability that the two red plates are adjacent and the two blue plates are adjacent.
- 3b. Three red and three blue plates are placed at a circular table for six people, with all arrangements of the plates equally likely. Find the probability that the three red plates are adjacent and the three blue plates are adjacent.
- 4a. Suppose a drawer has 5 pairs of socks (10 socks total), with each pair having its own unique color. If 3 socks are chosen, what is the probability that none of them match?
- 4b. Suppose a drawer has 8 pairs of socks (16 socks total), with each pair having its own unique color. If 5 socks are chosen, what is the probability that none of them match?
- 4c. In general, if a drawer has  $s$  pairs of socks ( $2s$  socks total), with each pair having its own unique color. If  $r$  socks are chosen (with  $r \leq s$ ), what is the probability that none of them match?
5. Six rocks are sitting in a straight line. We paint them, using up to three colors (say,  $R$ 's,  $W$ 's, and  $B$ 's). Suppose all of the  $3^6 = 729$  outcomes are equally likely.
  - 5a. Find the probability that exactly 1 color of paint is used.
  - 5b. Find the probability that exactly 2 colors of paint are used.
  - 5c. Find the probability that all 3 colors of paint are used.  
[Hint: Your answers should sum to 1 altogether.]
- 6a. Simplify the expression  $\sum_{j=3}^{\infty} a^j$ , where  $0 \leq a < 1$ .
- 6b. Simplify the expression  $\sum_{j=r}^{\infty} a^j$ , where  $0 \leq a < 1$ , and  $r$  is a fixed, positive constant.