1a. Let $A_j$ denote the event that the $j$th die has value 2. Then $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = 1/6 + 1/6 + 1/6 - (1/6)^2 - (1/6)^2 - (1/6)^2 + (1/6)^3 = 91/216$. [[To double-check: The probability a specific die does not show 2 is 5/6. The probability none of the three dice show 2 is (5/6)^3. So the probability that at least one of the three dice show 2 is 1 - (5/6)^3 = 91/216.]]

1b. The probability that no 2’s appear is (5/6)^3. For exactly one value of 2, there are three disjoint ways to choose which die it will appear on, and then the probability is (5/6)^2(1/6).

2a. Always true!! By inclusion-exclusion, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, but $P(A \cap B) \leq 0$, so $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, but $P(A \cap B) \geq 0$, so $P(A \cup B) = P(A) + P(B) - (P(A \cap B) \leq P(A) + P(B)$.

3a. If we place one red plate on the table, the second red plate has 3 equally likely places it could go, 2 of which are adjacent to the first red plate. So the red plates are adjacent with probability 2/3 (and the blue plates will automatically be adjacent in such a case).

3b. Without loss of generality, place one red plate on the table. If all three red plates are to be in a cluster, this first red plate could be the far left, the middle, or the far right of the three in the eventual cluster (i.e., 3 possibilities). Each such possibility has probability (2/5)(1/4) = 2/20 of occurring. So the total probability is 2/20 + 2/20 + 2/20 = 6/20 = 3/10. [[Alternative method: Place one red plate on the table. Then there are (3) = 10 equally likely ways remaining for the blue plates; in 3 of these ways, the blue plates are adjacent and therefore the red plates are adjacent too, so the probability is 3/10.]]

4a. There are (10)(9)(8) = 10 equally likely ways to pick, keeping track of order. Exactly (10)(8)(6) = (2^3)(5)(4)(3) = (2^3)5!/2! ways yield unique colors. So the probability is (2^3)5!7!/(2^10+2!10!) = 2/3. [[Without keeping track of order, the probability is again (3)2^3/(10) = 2/3.]]


4c. There are (2s)(2s−1)⋯(2s−r+1) = (2s)!/(2s−r)! equally likely ways to pick r socks, keeping track of order. Exactly (2s)(2s−2)⋯(2s−2(r−1)) = (2^r)(s−1)⋯(s−r+1) = (2^r)s!/(s−r)! ways yield unique colors. So the probability (of getting r uniquely colored socks) is (2^r)s!/(s−r)!

5a. There are 3 equally likely ways to paint all rocks the same color; the probability is 3/729.

5b. There are 3 ways to pick two colors. With these two colors only, there are 2^6 = 64 ways to paint all the rocks, but 2 of these 64 ways only actually use 1 of the 2 colors. So there are 62 ways to use both of the 2 selected colors. So the desired probability is (3)(62)/729 = 186/729.

5c. Since the probabilities must sum to 1, the probability that all 3 colors of paint are used is 1 − 3/729 − 186/729 = 540/729.

6a. We have $\sum_{j=3}^{\infty} a^j = a^3 + a^4 + a^5 + a^6 + a^7 + \cdots = a^3(1 + a + a^2 + a^3 + a^4 + \cdots) = \frac{a^3}{1-a}$.

6b. We have $\sum_{j=r}^{\infty} a^j = a^r + a^{r+1} + a^{r+2} + a^{r+3} + \cdots = a^r(1 + a + a^2 + a^3 + a^4 + \cdots) = \frac{a^r}{1-a}$. 