STAT/MA 41600
In-Class Problem Set #3: September 3, 2014

1. Consider a sample space S with eight outcomes, $S = \{a, b, c, d, e, f, g, h\}$. Suppose that each outcome is equally likely to appear. Now define the events $A = \{a, b, c, d\}$ and $B = \{c, d, e, f\}$. Are A and B independent events? Please justify your answer.

2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob chooses 2 marbles. Let A denote the event that Alice's 2 marbles have a matching color. Let B denote the event that Bob's 2 marbles have a matching color. Are A and B independent events? Please justify your answer.

3. Doug is indecisive. Doug randomly chooses a marble from a drawer with r red, b blue, g green, and y yellow marbles (all choices are equally likely). If he chooses a green or blue marble, he goes to the movies. If he chooses a red marble, he (instead) goes to dinner. If he chooses a yellow marble, he replaces the yellow marble back in the drawer, and he tries again. The game continues until he decides what to do: a movie or dinner. (He cannot afford to do both.)

3a. What is the probability that he (eventually) chooses to go to dinner?

3b. What is the probability that he (eventually) chooses to go to the movies?

3c. What is the probability that 3 or more yellow marbles are chosen, before a decision is made?

4. Roll a (6-sided) red die and a (6-sided) green die. Let A denote the event that the sum of the dice is odd (i.e., the sum is 3, 5, 7, 9, or 11). Let B denote the event that the red die shows an odd value (i.e., the red die shows 1, 3, or 5). Are A and B independent events? Please justify your answer.

5. Roll a pair of dice, where one is a 4-sided die, and the other is a 6-sided die. Continue rolling the pair of both dice (together) until the sum of the dice are 2 (i.e., until both dice simultaneously show the value 1). Stop the game at this point, and quit.

Find the probability that the sum of the pair of dice is *never* 7 during the game.

6a. Suppose Alice flips 4 fair coins, and Bob (independently) flips 3 fair coins. What is the probability Alice gets *strictly more* heads than Bob? [[Hint: You can solve this problem either by a direct calculation; or, you can simply think about the status of the game after Alice has flipped only 3 of her coins, and then see what happens when the 4th flip occurs.]]

6b. Can you generalize? I.e., if Alice flips $n + 1$ fair coins and Bob (independently) flips n fair coins, what is the probability Alice gets *strictly more* heads than Bob? [[One of my colleagues created this interesting generalization.]]