

STAT/MA 41600
In-Class Problem Set #4: September 5, 2014

1. Roll a red die and a green die. Given that the result on the red die is less than or equal to the result on the green die, find the probability that the two results are equal.
2. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Alice chooses 2 marbles without replacement, and then Bob chooses 2 marbles. Let A denote the event that Alice's 2 marbles have a matching color. Let B denote the event that Bob's 2 marbles have a matching color. Find $P(B | A^c)$, i.e., given that Alice's marbles do *not* have a matching color, find the probability that Bob's marbles have a matching color.
3. Suppose that a box contains 10 balls. At the start, 3 are white and 7 are blue. Whenever a ball is selected from the box, a layer of blue paint is applied to it, so blue balls stay blue, and white balls become blue; afterward, the ball is returned to the box, so that 10 balls are always in the box.
 - 3a. Find the probability that the 6th ball to be drawn from the box is white.
 - 3b. Find the probability that the 16th ball to be drawn from the box is white.
 - 3c. Find the probability that the 76th ball to be drawn from the box is white.
- 4a. Consider 3 students who wear similar-looking black coats to a party. When they leave the party, they are in a hurry, and they each randomly grab a coat as they depart, with all possibilities equally likely. Use inclusion-exclusion to calculate the probability that none of them get their correct coat. [Hint: First find the probability that *at least one* of them gets their correct coat.]
- 4b. Find the probability that nobody gets their correct coat, if we change 3 to 5 throughout.
- 4c. Find the probability that nobody gets their correct coat, if we change 3 to 10 throughout.
- 4d. Find an approximation for the probability that nobody gets their correct coat, when a very large number of people attend the party.
5. In every round of a game, Alice and Bob roll a die simultaneously. Alice's die is 6-sided. Bob's die is 4-sided. Find the probability that the very first time Alice rolls a value "1," it happens during exactly the same time when Bob rolls a value "1" for the very first time too; i.e., find the probability that their very first 1's happen to arrive during the same round.
6. Roll three 6-sided dice. Given that at least one of the dice shows the value "2," find the probability that all three of the dice show the value "2."