

STAT/MA 41600
 In-Class Problem Set #7: September 10, 2014
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1. There are $6^3 = 216$ equally-likely possible outcomes. So **(1a)** we have $P(X = 0) = \binom{3}{0}(1/6)^0(5/6)^3 = \frac{125}{216}$; **(1b)** we have $P(X = 1) = \binom{3}{1}(1/6)^1(5/6)^2 = \frac{25}{72}$; **(1c)** we have $P(X = 2) = \binom{3}{2}(1/6)^2(5/6)^1 = \frac{5}{72}$; **(1d)** we have $P(X = 3) = \binom{3}{3}(1/6)^3(5/6)^0 = \frac{1}{216}$.

2. Method #1: We have $X = 0$ if Alice and Bob get no reds, so $P(X = 0) = \frac{\binom{6}{2}\binom{4}{2}}{\binom{8}{2}\binom{6}{2}} = 3/14$. We have $X = 1$ if Alice gets 1 red and Bob gets none (happens with probability $\frac{\binom{2}{1}\binom{6}{1}\binom{5}{2}}{\binom{8}{2}\binom{6}{2}} = 2/7$) or Bob gets 1 red and Alice gets none (again, probability $2/7$). So $P(X = 1) = 4/7$. Finally, $X = 2$ if Alice gets both reds (probability $\frac{\binom{2}{2}\binom{6}{2}}{\binom{8}{2}\binom{6}{2}} = 1/28$), or Bob gets both reds (prob. $1/28$), or they each get one red (probability $\frac{\binom{2}{1}\binom{6}{1}\binom{1}{1}\binom{5}{1}}{\binom{8}{2}\binom{6}{2}} = 1/7$). So $P(X = 2) = 3/14$.

Method #2: We can combine the choices of Alice and Bob. So $P(X = 0) = \binom{2}{0}\binom{6}{4}/\binom{8}{4} = 3/14$, and $P(X = 1) = \binom{2}{1}\binom{6}{3}/\binom{8}{4} = 4/7$, and $P(X = 2) = \binom{2}{2}\binom{6}{2}/\binom{8}{4} = 3/14$.

3. There are 24 equally likely outcomes. Exactly 9 yield a minimum of 1; and 7 yield a minimum of 2; and 5 yield a minimum of 3; and 3 yield a minimum of 4. So $P(X = 1) = 9/24$; $P(X = 2) = 7/24$; $P(X = 3) = 5/24$; $P(X = 4) = 3/24$.

4. There are 24 equally likely outcomes. Exactly 1 yields a maximum of 1; and 3 yield a maximum of 2; and 5 yield a maximum of 3; and 7 yield a maximum of 4; and 4 yield a maximum of 5; and 4 yield a maximum of 6. So $P(X = 1) = 1/24$; $P(X = 2) = 3/24$; $P(X = 3) = 5/24$; $P(X = 4) = 7/24$; $P(X = 5) = 4/24$; and $P(X = 6) = 4/24$.

5. We have **(5a)** $P(X = 7) = (7/10)(7/10) = 49/100$, and **(5b)** $P(X = 8) = (7/10)(3/10) + (3/10)(8/10) = 45/100$, and **(5c)** $P(X = 9) = (3/10)(2/10) = 6/100$.

6a. If 0 blues and 3 reds are chosen, then $X = 0$, so $P(X = 0) = \frac{\binom{3}{0}\binom{3}{3}}{\binom{6}{3}} = 1/20$.

6b. If 1 blue and 2 reds are chosen, then $X = 1$, so $P(X = 1) = \frac{\binom{3}{1}\binom{3}{2}}{\binom{6}{3}} = 9/20$.

6c. If 2 blue and 1 reds are chosen, then $X = 2$, so $P(X = 2) = \frac{\binom{3}{2}\binom{3}{1}}{\binom{6}{3}} = 9/20$.

6d. If 3 blues and 0 reds are chosen, then $X = 3$, so $P(X = 3) = \frac{\binom{3}{3}\binom{3}{0}}{\binom{6}{3}} = 1/20$.