STAT/MA 41600
In-Class Problem Set #8: September 12, 2014
Solutions by Mark Daniel Ward

1. The mass of X is $p_X(x) = (32/52)^{x-1}(20/52)$, for integers $x \geq 1$. So the CDF of X, for an integer $x \geq 1$, is $F_X(x) = \sum_{j=1}^{x}(32/52)^{j-1}(20/52) = (20/52)\frac{1-(32/52)^x}{1-32/52} = 1-(32/52)^x$.

2a. The mass of X is $p_X(x) = (5/6)^{x-1}(1/6)$, for integers $x \geq 1$. The probability X is even is $(5/6)^1(1/6)+(5/6)^3(1/6)+(5/6)^5(1/6)+(5/6)^7(1/6)+\cdots = (5/6)(1/6)(1+(5/6)^2+(5/6)^4+(5/6)^6+\cdots) = (5/6)(1/6)\frac{1}{1-(5/6)^2} = 5/11$.

2b. The probability Y is even is $pq+pq^3+pq^5+pq^7+\cdots = pq(1+q^2+q^4+q^6+\cdots) = \frac{pq}{1-q^2}$.

3a. The mass of X is $p_X(x) = (5/6)^{x-1}(1/6)$, for integers $x \geq 1$. The probability X is a multiple of three is $(5/6)^2(1/6)+(5/6)^5(1/6)+(5/6)^8(1/6)+(5/6)^{11}(1/6)+\cdots = (5/6)^2(1/6)(1+(5/6)^3+(5/6)^6+(5/6)^9+\cdots) = (5/6)^2(1/6)\frac{1}{1-(5/6)^3} = 25/91$.

3b. The probability Y is a multiple of 3 is $pq^2+pq^5+pq^8+pq^{11}+\cdots = pq^2(1+q^3+q^6+q^9+\cdots) = \frac{pq^2}{1-q^3}$.

4. The probability that they get the exact same number of heads is $(1/16)(1/16)+(4/16)(4/16)+(6/16)(6/16)+(4/16)(4/16)+(1/16)(1/16) = 35/128$.

5. The mass of X is: $p_X(1) = 7/28$, $p_X(2) = 6/28$, $p_X(3) = 5/28$, $p_X(4) = 4/28$, $p_X(5) = 3/28$, $p_X(6) = 2/28$, and $p_X(7) = 1/28$.

6. The mass of X is $p_X(0) = \frac{\binom{4}{0}\binom{4}{4}}{\binom{8}{4}} = 1/70$, $p_X(1) = \frac{\binom{4}{1}\binom{4}{3}}{\binom{8}{4}} = 8/35$, $p_X(2) = \frac{\binom{4}{2}\binom{4}{2}}{\binom{8}{4}} = 18/35$, $p_X(3) = \frac{\binom{4}{3}\binom{4}{1}}{\binom{8}{4}} = 8/35$, $p_X(4) = \frac{\binom{4}{4}\binom{4}{0}}{\binom{8}{4}} = 1/70$. So the CDF of X, as $x = 0,1,2,3,4$, is $F_X(0) = 1/70$, $F_X(1) = 17/70$, $F_X(2) = 53/70$, $F_X(3) = 69/70$, $F_X(4) = 1$.