

STAT/MA 41600
In-Class Problem Set #9: September 15, 2014

1. Suppose Alice and Bob each take a cookie, without replacement, from a jar that contains 5 cookies, 3 of which are chocolate, and the other 2 are non-chocolate. Let $X = 1$ if Alice gets chocolate; let $X = 0$ otherwise. Let $Y = 1$ if Bob gets chocolate; let $Y = 0$ otherwise.

- 1a.** Find the joint probability mass functions $p_{X,Y}(x, y)$ of X and Y .
- 1b.** Find the conditional probability mass function $p_{X|Y}(x|1)$ of X , given $Y = 1$.
- 1c.** Find the conditional probability mass function $p_{X|Y}(x|0)$ of X , given $Y = 0$.
- 1d.** Are X and Y independent? Why or why not?

2. Roll two dice. Let X be the number of 5's that appear. Let Y be the maximum of the values appearing on the two dice. Find the joint probability mass function of X and Y . [Hint: there are only eight values of the pair (x, y) for which $p_{X,Y}(x, y)$ is strictly positive.]

3. Suppose that X and Y are independent random variables, such that X has mass $p_X(x) = (3/5)(2/5)^{x-1}$ for integers $x \geq 1$, and Y has mass $p_Y(y) = (7/10)(3/10)^{y-1}$ for integers $y \geq 1$.

- 3a.** Find $P(X = Y)$.
- 3b.** Find $P(X > Y)$.
- 3c.** Find $P(X < Y)$.

4. Roll a 4-sided die. Whatever value appears, flip exactly that many fair coins. [For instance, if the die shows 3, then flip 3 fair coins.] Let X denote the number of heads that appear on the coins; let Y denote the number of tails that appear on the coins. Find the mass $p_{X,Y}(x, y)$ for all integers $x \geq 0$ and $y \geq 0$ that satisfy $1 \leq x + y \leq 4$. [Hint: There are only fourteen such values of the pair (x, y) .]

5. Suppose that X and Y have joint mass $p_{X,Y}(x, y) = \frac{xy}{65}$ for positive integers x, y that satisfy $1 \leq y \leq x \leq 4$.

- 5a.** Verify that this is indeed a joint mass, i.e., that the values of $p_{X,Y}(x, y)$ are all nonnegative and that they sum to 1.
- 5b.** Find the mass of X , i.e., find $p_X(x)$, for integers $1 \leq x \leq 4$.
- 5c.** Find the mass of Y , i.e., find $p_Y(y)$, for integers $1 \leq y \leq 4$.

6. Suppose that X and Y have joint mass $p_{X,Y}(x, y) = (1/2)^x(2/3)^y$ for positive integers x, y that satisfy $1 \leq y \leq x$.

- 6a.** Verify that this is indeed a joint mass, i.e., that the values of $p_{X,Y}(x, y)$ are all nonnegative and that they sum to 1.
- 6b.** Find the mass of X , i.e., find $p_X(x)$, for integers $x \geq 1$.
- 6c.** Find the mass of Y , i.e., find $p_Y(y)$, for integers $y \geq 1$.