

STAT/MA 41600
In-Class Problem Set #9: September 15, 2014
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1a. The joint mass is $p_{X,Y}(1,1) = (3/5)(2/4) = 3/10$, $p_{X,Y}(1,0) = (3/5)(2/4) = 3/10$, $p_{X,Y}(0,1) = (2/5)(3/4) = 3/10$, and $p_{X,Y}(0,0) = (2/5)(1/4) = 1/10$.

1b. We have $p_Y(1) = p_{X,Y}(1,1) + p_{X,Y}(0,1) = 3/10 + 3/10 = 3/5$. So the conditional mass of X given $Y = 1$ is $p_{X|Y}(1|1) = \frac{p_{X,Y}(1,1)}{p_Y(1)} = \frac{3/10}{3/5} = 1/2$, and $p_{X|Y}(0|1) = \frac{p_{X,Y}(0,1)}{p_Y(1)} = \frac{3/10}{3/5} = 1/2$.

1c. We have $p_Y(0) = p_{X,Y}(1,0) + p_{X,Y}(0,0) = 3/10 + 1/10 = 2/5$. So the conditional mass of X given $Y = 0$ is $p_{X|Y}(1|0) = \frac{p_{X,Y}(1,0)}{p_Y(0)} = \frac{3/10}{2/5} = 3/4$, and $p_{X|Y}(0|0) = \frac{p_{X,Y}(0,0)}{p_Y(0)} = \frac{1/10}{2/5} = 1/4$.

1d. The random variables X and Y are not independent. For instance, $p_X(1) = p_{X,Y}(1,0) + p_{X,Y}(1,1) = 3/10 + 3/10 = 3/5$, but $p_{X|Y}(1|0) = 3/4$. Since $p_X(1) \neq p_{X|Y}(1|0)$, then X and Y are dependent.

2. We have $p_{X,Y}(2,5) = 1/36$, $p_{X,Y}(1,6) = 2/36$, $p_{X,Y}(1,5) = 8/36$, $p_{X,Y}(0,6) = 9/36$, $p_{X,Y}(0,4) = 7/36$, $p_{X,Y}(0,3) = 5/36$, $p_{X,Y}(0,2) = 3/36$, and $p_{X,Y}(0,1) = 1/36$.

3a. We have $P(X = Y) = \sum_{n=1}^{\infty} P(X = Y = n) = \sum_{n=1}^{\infty} P(X = n)P(Y = n) = \sum_{n=1}^{\infty} (3/5)(2/5)^{n-1}(7/10)(3/10)^{n-1} = \frac{(3/5)(7/10)}{1-(2/5)(3/10)} = 21/44$.

3b. We have $P(X > Y) = \sum_{n=1}^{\infty} P(X > Y = n) = \sum_{n=1}^{\infty} P(X > n)P(Y = n) = \sum_{n=1}^{\infty} (2/5)^n(7/10)(3/10)^{n-1} = \frac{(2/5)(7/10)}{1-(2/5)(3/10)} = 14/44$.

3c. We have $P(Y > X) = \sum_{n=1}^{\infty} P(Y > X = n) = \sum_{n=1}^{\infty} P(Y > n)P(X = n) = \sum_{n=1}^{\infty} (3/10)^n(3/5)(2/5)^{n-1} = \frac{(3/5)(3/10)}{1-(2/5)(3/10)} = 9/44$.

4. We have $p_{X,Y}(0,1) = p_{X,Y}(1,0) = (1/4)(1/2) = 1/8$;
 $p_{X,Y}(0,2) = p_{X,Y}(2,0) = (1/4)(1/4) = 1/16$ and $p_{X,Y}(1,1) = (1/4)(1/2) = 1/8$;
 $p_{X,Y}(0,3) = p_{X,Y}(3,0) = (1/4)(1/8) = 1/32$ and $p_{X,Y}(1,2) = p_{X,Y}(2,1) = (1/4)(3/8) = 3/32$;
 $p_{X,Y}(0,4) = p_{X,Y}(4,0) = (1/4)(1/16) = 1/64$ and $p_{X,Y}(1,3) = p_{X,Y}(3,1) = (1/4)(4/16) = 1/16$ and $p_{X,Y}(2,2) = (1/4)(6/16) = 3/32$.

5a. The mass sums to 1 because $\frac{(1)(1)}{65} + \frac{(2)(1)}{65} + \frac{(2)(2)}{65} + \frac{(3)(1)}{65} + \frac{(3)(2)}{65} + \frac{(3)(3)}{65} + \frac{(4)(1)}{65} + \frac{(4)(2)}{65} + \frac{(4)(3)}{65} + \frac{(4)(4)}{65} = 1$.

5b. The mass of X is: $p_X(1) = 1/65$; $p_X(2) = 2/65 + 4/65 = 6/65$; $p_X(3) = 3/65 + 6/65 + 9/65 = 18/65$; $p_X(4) = 4/65 + 8/65 + 12/65 + 16/65 = 40/65$.

5c. The mass of Y is: $p_Y(1) = 1/65 + 2/65 + 3/65 + 4/65 = 10/65$; $p_Y(2) = 4/65 + 6/65 + 8/65 = 18/65$; $p_Y(3) = 9/65 + 12/65 = 21/65$; $p_Y(4) = 16/65$.

6a. The mass sums to 1 because $\sum_{y=1}^{\infty} \sum_{x=y}^{\infty} (1/2)^x(2/3)^y = \sum_{y=1}^{\infty} (2/3)^y \sum_{x=y}^{\infty} (1/2)^x = \sum_{y=1}^{\infty} (2/3)^y \frac{(1/2)^y}{1-1/2} = 2 \sum_{y=1}^{\infty} (1/3)^y = \frac{2(1/3)}{1-1/3} = 1$.

6b. The mass of X , for integers $x \geq 1$, is $p_X(x) = \sum_{y=1}^x (1/2)^x(2/3)^y = (1/2)^{x-1} - 2(1/3)^x$.

6c. The mass of Y , for integers $y \geq 1$, is $p_Y(y) = \sum_{x=y}^{\infty} (1/2)^x(2/3)^y = 2(1/3)^y$.