

STAT/MA 41600
In-Class Problem Set #10: September 17, 2014
Solutions by Mark Daniel Ward

1. Use X to denote the number of selected students who live on-campus. Then the mass of X is: $P(X = 0) = \binom{6}{0}(.40)^0(.60)^6$; $P(X = 1) = \binom{6}{1}(.40)^1(.60)^5$; $P(X = 2) = \binom{6}{2}(.40)^2(.60)^4$; $P(X = 3) = \binom{6}{3}(.40)^3(.60)^3$; $P(X = 4) = \binom{6}{4}(.40)^4(.60)^2$; $P(X = 5) = \binom{6}{5}(.40)^5(.60)^1$; $P(X = 6) = \binom{6}{6}(.40)^6(.60)^0$; so $\mathbb{E}(X) = 0P(X = 0) + 1P(X = 1) + \cdots + 6P(X = 6) = 12/5$.

2. The probability mass of X is: $P(X = 0) = \binom{7}{0}(1/3)^0(2/3)^7$; $P(X = 1) = \binom{7}{1}(1/3)^1(2/3)^6$; $P(X = 2) = \binom{7}{2}(1/3)^2(2/3)^5$; $P(X = 3) = \binom{7}{3}(1/3)^3(2/3)^4$; $P(X = 4) = \binom{7}{4}(1/3)^4(2/3)^3$; $P(X = 5) = \binom{7}{5}(1/3)^5(2/3)^2$; $P(X = 6) = \binom{7}{6}(1/3)^6(2/3)^1$; $P(X = 7) = \binom{7}{7}(1/3)^7(2/3)^0$; so $\mathbb{E}(X) = 0P(X = 0) + 1P(X = 1) + \cdots + 7P(X = 7) = 7/3$.

3a. The mass of X is: $P(X = 1) = \frac{\binom{3}{1}\binom{2}{2}}{\binom{5}{3}}$; $P(X = 2) = \frac{\binom{3}{2}\binom{2}{1}}{\binom{5}{3}}$; $P(X = 3) = \frac{\binom{3}{3}\binom{2}{0}}{\binom{5}{3}}$; so $\mathbb{E}(X) = 1P(X = 1) + 2P(X = 2) + 3P(X = 3) = 9/5$.

3b. The mass of Y is different: $P(Y = 0) = \binom{3}{0}(3/5)^0(2/5)^3$; $P(Y = 1) = \binom{3}{1}(3/5)^1(2/5)^2$; $P(Y = 2) = \binom{3}{2}(3/5)^2(2/5)^1$; $P(Y = 3) = \binom{3}{3}(3/5)^3(2/5)^0$; so $\mathbb{E}(X) = 0P(X = 0) + 1P(X = 1) + 2P(X = 2) + 3P(X = 3) = 9/5$.

4. As in Problem Set #8, we know that the mass of X is: $p_X(1) = 7/28$, $p_X(2) = 6/28$, $p_X(3) = 5/28$, $p_X(4) = 4/28$, $p_X(5) = 3/28$, $p_X(6) = 2/28$, and $p_X(7) = 1/28$, so $\mathbb{E}(X) = 1P(X = 1) + 2P(X = 2) + \cdots + 7P(X = 7) = 3$.

5. As in Problem Set #7, the mass of X is: $P(X = 1) = 9/24$; $P(X = 2) = 7/24$; $P(X = 3) = 5/24$; $P(X = 4) = 3/24$, so $\mathbb{E}(X) = 1P(X = 1) + 2P(X = 2) + 3P(X = 3) + 4P(X = 4) = 25/12$.

6. As in Problem Set #2, we know that the mass of X is: $P(X = 1) = 3/729$; $P(X = 2) = 186/729$; $P(X = 3) = 540/729$, so $\mathbb{E}(X) = 1P(X = 1) + 2P(X = 2) + 3P(X = 3) = 665/243$.