

STAT/MA 41600
In-Class Problem Set #11: September 19, 2014

Solve all problems by decomposing the random variable in each problem as a sum of indicator random variables. In other words, find indicator random variables X_1, X_2, X_3, \dots such that $X = X_1 + X_2 + X_3 + \dots$, and use the fact that $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3 + \dots) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \dots$ to find the expected value.

1. At a certain college, 40% of the students live in a residence hall (on-campus), and the other 60% of the students live off-campus. If 6 students are independently selected at random, find the expected number of those students who live in a residence hall (on-campus).

2. When rolling a die, a “high value” is a 5 or 6. Roll seven dice. Let X denote the number of “high values” obtained on the seven dice altogether. Find $\mathbb{E}(X)$.

3a. Suppose Alice takes 3 cookies (without replacement) from a cookie jar that contains 5 cookies, 3 of which are chocolate, and the other 2 are non-chocolate. Let X be the number of chocolate cookies she gets. Find $\mathbb{E}(X)$.

3b. Same type of question, but with replacement: Suppose Alice takes 3 cookies (with replacement) from a cookie jar that contains 5 cookies, 3 of which are chocolate, and the other 2 are non-chocolate. Let Y be the number of chocolate cookies she gets. Find $\mathbb{E}(Y)$.

4. Suppose that a drawer contains 8 marbles: 2 are red, 2 are blue, 2 are green, and 2 are yellow. The marbles are rolling around in a drawer, so that all possibilities are equally likely when they are drawn. Suppose that a person removes marbles from the drawer, one at a time, without replacement, and she stops when red is selected for the first time. Let X denote the number of marbles removed, until red is selected for the first time. Find the expected value of X .

5. Suppose Alice rolls a 6-sided die, and Bob rolls a 4-sided die. Let X denote the *minimum* value on the two dice. Find $\mathbb{E}(X)$.

6. Six rocks are sitting in a straight line. We paint them, using up to three colors (say, R 's, W 's, and B 's). Suppose all of the $3^6 = 729$ outcomes are equally likely. Let X denote the number of colors used altogether to paint the rocks. Find $\mathbb{E}(X)$.