STAT/MA 41600
In-Class Problem Set #11: September 19, 2014
Solutions by Mark Daniel Ward

1. Use X to denote the number of selected students who live on-campus. Let $X_1, X_2, \ldots, X_6$ denote (respectively) whether the 1st, 2nd, …, 6th person lives on-campus. Then $X = X_1 + \cdots + X_6$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_6) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_6) = 2/5 + 2/5 + 2/5 + 2/5 + 2/5 + 2/5 = 12/5$.

2. Let $X_1, X_2, \ldots, X_7$ denote (respectively) whether the 1st, 2nd, …, 7th die shows a "high value". Then $X = X_1 + \cdots + X_7$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_7) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_7) = 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 + 1/3 = 7/3$.

3a. Let X_1, X_2, X_3 denote (respectively) whether the 1st, 2nd, 3rd cookie chosen is chocolate. Then $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 3/5 + 3/5 + 3/5 = 9/5$.
3b. Let Y_1, Y_2, Y_3 denote (respectively) whether the 1st, 2nd, 3rd cookie chosen is chocolate. Then $Y = Y_1 + Y_2 + Y_3$, so $\mathbb{E}(Y) = \mathbb{E}(Y_1 + Y_2 + Y_3) = \mathbb{E}(Y_1) + \mathbb{E}(Y_2) + \mathbb{E}(Y_3) = 3/5 + 3/5 + 3/5 = 9/5$.

4. Let $X_1 = 1$ if $X \geq 1$, or $X_1 = 0$ otherwise. Let $X_2 = 1$ if $X \geq 2$, or $X_2 = 0$ otherwise. Etc., etc. In general, let Let $X_j = 1$ if $X \geq j$, or $X_j = 0$ otherwise. Then $X = X_1 + \cdots + X_7$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_7) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_7) = 1 + (6/8) + (6/8)(5/7) + (6/8)(5/7)(4/6) + (6/8)(5/7)(4/6)(3/5) + (6/8)(5/7)(4/6)(3/5)(2/4) + (6/8)(5/7)(4/6)(3/5)(2/4)(1/3) = 3$.

5. Let $X_1 = 1$ if $X \geq 1$, or $X_1 = 0$ otherwise. Let $X_2 = 1$ if $X \geq 2$, or $X_2 = 0$ otherwise. Let $X_3 = 1$ if $X \geq 3$, or $X_3 = 0$ otherwise. Let $X_4 = 1$ if $X \geq 4$, or $X_4 = 0$ otherwise. Then $X = X_1 + X_2 + X_3 + X_4$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + \cdots + X_4) = \mathbb{E}(X_1) + \cdots + \mathbb{E}(X_4) = 1 + (5/6)(3/4) + (4/6)(2/4) + (3/6)(1/4) = 25/12$.

6. Let $X_1 = 1$ if red is ever used, or $X_1 = 0$ otherwise. Let $X_2 = 1$ if white is ever used, or $X_2 = 0$ otherwise. Let $X_3 = 1$ if blue is ever used, or $X_3 = 0$ otherwise. Then $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = \frac{3^6 - 2^6}{3^6} + \frac{3^6 - 2^6}{3^6} + \frac{3^6 - 2^6}{3^6} = 665/243$.