1a. The mass of X is $P(X = j) = \binom{6}{j}(.40)^j(.60)^{6-j}$ for $0 \le j \le 6$, so $\mathbb{E}(X^2) = 0^2 P(X = 0) + 1^2 P(X = 1) + \cdots + 6^2 P(X = 6) = 36/5$.

1b. Let $X_1, X_2, \ldots, X_6$ denote (respectively) whether the 1st, 2nd, $\ldots$, 6th person lives on-campus. Then $X = X_1 + \cdots + X_6$, so $\mathbb{E}(X^2) = \mathbb{E}((X_1 + \cdots + X_6)^2)$, which has 30 terms of the form $\mathbb{E}(X_i X_j)$ (for $i \ne j$) and 6 terms of the form $\mathbb{E}(X_j^2)$. Since X_i and X_j are independent for $i \ne j$, then $\mathbb{E}(X_i X_j) = \mathbb{E}(X_i)\mathbb{E}(X_j) = (.4)(.4) = .16$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2) = \mathbb{E}(X_j) = .4$. Thus $\mathbb{E}(X^2) = (30)(.16) + (6)(.4) = 36/5$.

1c. We have $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 36/5 - (12/5)^2 = 36/25$.

1d. Since the X_j's are independent, $\text{Var}(X) = \text{Var}(X_1 + \cdots + X_6) = \text{Var}(X_1) + \cdots + \text{Var}(X_6)$. We have $\text{Var}(X_j) = \mathbb{E}(X_j^2) - (\mathbb{E}(X_j))^2 = .4 - (.4)^2 = 6/25$, so $\text{Var}(X) = 6(6/25) = 36/25$.

2a. The mass of X is $P(X = j) = \binom{7}{j}(1/3)^j(2/3)^{7-j}$ for $0 \le j \le 7$, so $\mathbb{E}(X^2) = 0^2 P(X = 0) + 1^2 P(X = 1) + \cdots + 7^2 P(X = 7) = 7$.

2b. Let $X_1, X_2, \ldots, X_7$ denote (respectively) whether the 1st, 2nd, $\ldots$, 7th die shows a "high value". Then $X = X_1 + \cdots + X_7$, so $\mathbb{E}(X^2) = \mathbb{E}((X_1 + \cdots + X_7)^2)$, which has 42 terms of the form $\mathbb{E}(X_i X_j)$ (for $i \ne j$) and 7 terms of the form $\mathbb{E}(X_j^2)$. Since X_i and X_j are independent for $i \ne j$, then $\mathbb{E}(X_i X_j) = \mathbb{E}(X_i)\mathbb{E}(X_j) = (1/3)(1/3) = 1/9$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2) = \mathbb{E}(X_j) = 1/3$. Thus $\mathbb{E}(X^2) = (42)(1/9) + (7)(1/3) = 7$.

2c. We have $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 7 - (7/3)^2 = 14/9$.

2d. Since the X_j's are independent, $\text{Var}(X) = \text{Var}(X_1 + \cdots + X_7) = \text{Var}(X_1) + \cdots + \text{Var}(X_7)$. We have $\text{Var}(X_j) = \mathbb{E}(X_j^2) - (\mathbb{E}(X_j))^2 = 1/3 - (1/3)^2 = 2/9$, so $\text{Var}(X) = 7(2/9) = 14/9$.

3a. The mass of X is $P(X = j) = \frac{\binom{3}{j}\binom{2}{3-j}}{\binom{5}{3}}$ for $1 \le j \le 3$, so $\mathbb{E}(X^2) = 1^2 P(X = 1) + 2^2 P(X = 2) + 3^2 P(X = 3) = 18/5$.

3b. Let X_1, X_2, X_3 denote (respectively) whether the 1st, 2nd, 3rd cookie chosen is chocolate. Then $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X^2) = \mathbb{E}((X_1 + X_2 + X_3)^2)$, which has 6 terms of the form $\mathbb{E}(X_i X_j)$ (for $i \ne j$) and 3 terms of the form $\mathbb{E}(X_j^2)$. We have $\mathbb{E}(X_i X_j) = (3/5)(2/4) = 3/10$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2) = \mathbb{E}(X_j) = 3/5$. Thus $\mathbb{E}(X^2) = (6)(3/10) + (3)(3/5) = 18/5$.

3c. We have $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 18/5 - (9/5)^2 = 9/25$.

3d. The mass of Y is $P(Y = j) = \binom{3}{j}(3/5)^j(2/5)^{3-j}$ for $0 \le j \le 3$, so $\mathbb{E}(X^2) =.$ so $\mathbb{E}(Y^2) = 0^2 P(Y = 0) + 1^2 P(Y = 1) + \cdots + 3^2 P(X = 3) = 99/25$.

3e. Let Y_1, Y_2, Y_3 denote (respectively) whether the 1st, 2nd, 3rd cookie chosen is chocolate. Then $X = Y_1 + Y_2 + Y_3$, so $\mathbb{E}(Y^2) = \mathbb{E}((Y_1 + Y_2 + Y_3)^2)$, which has 6 terms of the form $\mathbb{E}(Y_i Y_j)$ (for $i \ne j$) and 3 terms of the form $\mathbb{E}(Y_j^2)$. Since Y_i and Y_j are independent for $i \ne j$, then $\mathbb{E}(Y_i Y_j) = \mathbb{E}(Y_i)\mathbb{E}(Y_j) = (3/5)(3/5) = 9/25$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(Y_j^2) = \mathbb{E}(Y_j) = 3/5$. Thus $\mathbb{E}(Y^2) = (6)(9/25) + (3)(3/5) = 99/25$.

3f. We have $\text{Var}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 = 99/25 - (9/5)^2 = 18/25$.

3g. Since the Y_j's are independent, $\text{Var}(Y) = \text{Var}(Y_1 + Y_2 + Y_3) = \text{Var}(Y_1) + \text{Var}(Y_2) + \text{Var}(Y_3)$. We have $\text{Var}(Y_j) = \mathbb{E}(Y_j^2) - (\mathbb{E}(Y_j))^2 = 3/5 - (3/5)^2 = 6/25$, so $\text{Var}(Y) = 3(6/25) = 18/25$.

4a. The mass of X is $p_X(1) = 7/28$, $p_X(2) = 6/28$, $p_X(3) = 5/28$, $p_X(4) = 4/28$, $p_X(5) = 3/28$, $p_X(6) = 2/28$, and $p_X(7) = 1/28$, so $\mathbb{E}(X^2) = 1^2 P(X = 1) + 2^2 P(X = 2) + \cdots + 7^2 P(X = 7) = 12$.

4b. In $\mathbb{E}(X^2)$, there are 12 terms of the form $\mathbb{E}(X_i X_7)$ where $i < 7$, but $X_i X_7 = 1$ if and only if $X \geq 7$, so $\mathbb{E}(X_i X_7) = \mathbb{E}(X_7) = 1/28$.

There are 10 terms of the form $\mathbb{E}(X_i X_6)$ where $i < 6$, but $X_i X_6 = 1$ if and only if $X \geq 6$, so $\mathbb{E}(X_i X_6) = \mathbb{E}(X_6) = 1/28 + 2/28 = 3/28$.

Etc., etc. Also, $\mathbb{E}(X_j^2) = \mathbb{E}(X_j)$ for each j. Thus

$$
\begin{aligned}
\mathbb{E}(X^2) &= (12)\mathbb{E}(X_7) + (10)\mathbb{E}(X_6) + (8)\mathbb{E}(X_5) + (6)\mathbb{E}(X_4) + (4)\mathbb{E}(X_3) + (2)\mathbb{E}(X_2) \\
&\quad + \mathbb{E}(X_7) + \mathbb{E}(X_6) + \mathbb{E}(X_5) + \mathbb{E}(X_4) + \mathbb{E}(X_3) + \mathbb{E}(X_2) + \mathbb{E}(X_1) \\
&= (13)\mathbb{E}(X_7) + (11)\mathbb{E}(X_6) + (9)\mathbb{E}(X_5) + (7)\mathbb{E}(X_4) + (5)\mathbb{E}(X_3) + (3)\mathbb{E}(X_2) + (1)\mathbb{E}(X_1) \\
&= (13)(1/28) + (11)(3/28) + (9)(6/28) + (7)(10/28) + (5)(15/28) + (3)(21/28) + (1)(1) \\
&= 12
\end{aligned}
$$

4c. We have $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 12 - (3)^2 = 3$.

5a. The mass of X is $P(X = 1) = 9/24$; $P(X = 2) = 7/24$; $P(X = 3) = 5/24$; $P(X = 4) = 3/24$, so $\mathbb{E}(X^2) = 1^2 P(X = 1) + 2^2 P(X = 2) + 3^2 P(X = 3) + 4^2 P(X = 4) = 65/12$.

5b. In $\mathbb{E}(X^2)$, there are 6 terms of the form $\mathbb{E}(X_i X_4)$ where $i < 4$, but $X_i X_4 = 1$ if and only if $X \geq 4$, so $\mathbb{E}(X_i X_4) = \mathbb{E}(X_4) = 3/24$. There are 4 terms of the form $\mathbb{E}(X_i X_3)$ where $i < 3$, but $X_i X_3 = 1$ if and only if $X \geq 3$, so $\mathbb{E}(X_i X_3) = \mathbb{E}(X_3) = 3/24 + 5/24 = 8/24$. There are 2 terms of the form $\mathbb{E}(X_1 X_2)$, but $X_1 X_2 = 1$ if and only if $X \geq 2$, so $\mathbb{E}(X_1 X_2) = \mathbb{E}(X_2) = 3/24 + 5/24 + 7/24 = 15/24$. Also, $\mathbb{E}(X_j^2) = \mathbb{E}(X_j)$ for each j. Thus

$$
\begin{aligned}
\mathbb{E}(X^2) &= (6)\mathbb{E}(X_4) + (4)\mathbb{E}(X_3) + (2)\mathbb{E}(X_2) + \mathbb{E}(X_4) + \mathbb{E}(X_3) + \mathbb{E}(X_2) + \mathbb{E}(X_1) \\
&= (7)\mathbb{E}(X_4) + (5)\mathbb{E}(X_3) + (3)\mathbb{E}(X_2) + (1)\mathbb{E}(X_1) \\
&= (7)(3/24) + (5)(8/24) + (3)(15/24) + (1)(1) \\
&= 65/12
\end{aligned}
$$

5c. We have $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 65/12 - (25/12)^2 = 155/144$.

6a. The mass of X is $P(X = 1) = 3/729$; $P(X = 2) = 186/729$; $P(X = 3) = 540/729$, so $\mathbb{E}(X^2) = 1^2 P(X = 1) + 2^2 P(X = 2) + 3^2 P(X = 3) = 623/81$.

6b. Let X_1, X_2, X_3 indicate (respectively) whether red, white, and blue is ever used. Then $X = X_1 + X_2 + X_3$, so $\mathbb{E}(X^2) = \mathbb{E}((X_1 + X_2 + X_3)^2)$, which has 6 terms of the form $\mathbb{E}(X_i X_j)$ (for $i \neq j$) and 3 terms of the form $\mathbb{E}(X_j^2)$. We have $\mathbb{E}(X_i X_j) = \frac{3^6 - 2^6 - 2^6 + 1^6}{3^6}$. Also, since indicators only take on values 0 or 1, then $\mathbb{E}(X_j^2) = \mathbb{E}(X_j) = \frac{3^6 - 2^6}{3^6}$. Thus $\mathbb{E}(X^2) = (6)\frac{3^6 - 2^6 - 2^6 + 1^6}{3^6} + (3)\frac{3^6 - 2^6}{3^6} = 623/81$.

6c. We have $\mathrm{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 623/81 - (665/243)^2 = 11942/59049$.