STAT/MA 41600
In-Class Problem Set #14/#15: September 24, 2014
(there is no Problem Set #13)

1. At a certain college, 40% of the students live in a residence hall (on-campus), and the other 60% of the students live off-campus. Suppose we interview students until we find the *first student* who lives on-campus. Let X be the number of interviews needed. Then $X = X_1 + X_2 + X_3 + \cdots$, where $X_j = 1$ if j or more interviews are needed; otherwise $X_j = 0$.

1a. Find the value of $\mathbb{E}(X_j)$, for each $j \geq 1$.

1b. Sum the values from part **(1a)** to obtain $\mathbb{E}(X)$.

2. When rolling a die, a "high value" is a 5 or 6. Suppose we roll dice until we obtain the *first "high value"*. Let X denote the number of rolls needed. Then $X = X_1 + X_2 + X_3 + \cdots$, where $X_j = 1$ if j or more rolls are needed, or $X_j = 0$ otherwise.

2a. Find the value of $\mathbb{E}(X_j)$, for each $j \geq 1$.

2b. Sum the values from part **(2a)** to obtain $\mathbb{E}(X)$.

3. Suppose Alice owns a circular table with chairs sitting around the table. She walks around the table, flipping a fair coin at each place, and writing the result (H or T, for Heads or Tails) on each chair, as she walks around the table.

When Bob gets home, he checks to see how many of the adjacent pairs of coin results match (i.e., how many times that adjacent chairs show the same result). Let X denote the total number of adjacent pairs of coin results that match.

3a. If Alice has 5 chairs around the table, then $X \leq 5$. Question: is X a Binomial random variable with $n = 5$ and $p = 1/2$? Why or why not? [[Hint: what values can X be?]]

3b. If Alice has 5 chairs around the table, find $\mathbb{E}(X)$.

3c. If Alice has 75 chairs around the (very large) table, find $\mathbb{E}(X)$.

4. Six rocks are sitting *in a circle* (not in a row). We paint them, using up to three colors (say, R's, W's, and B's). Suppose all of the $3^6 = 729$ outcomes are equally likely. Let X denote the number of times that adjacent rocks have the same color. Find $\mathbb{E}(X)$.

5. Let X and Y denote two independent die rolls, and $Z = |X - Y|$ is their difference (in absolute value, i.e., $Z \geq 0$). Find $\mathbb{E}(Z)$.

6. On each round, Alice and Bob independent (and simultaneously) roll a die. They quit the game when their dice show the same value. During the game, let X denote the number of times that the sum of the two dice (during that round) was 9. Let Y denote the number of times that the sum of the two dice (during that round) was 11.

For instance, if they roll the following, then $X = 4$ and $Y = 3$ in this particular case.

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Alice:  4 6 3 3 1 6 6 6 2 2 2 5 6 3 3 2 5 3 2
  Bob:  1 2 6 2 4 3 5 3 5 4 1 6 5 2 2 1 4 4 2
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In general (i.e., not in the example above), suppose that $X + Y = 5$. Find the conditional probability mass function of X, given that $X + Y = 5$.