1. Matilda rolls a die until the first occurrence of “1,” and then she stops. Let $X$ denote the number of rolls until (and including) that first occurrence of “1”; let $Y$ denote the number of rolls that occur strictly before (but not including) that “1.” So we always have $Y = X - 1$.
1a. Find $E(X)$.
1b. Find $\text{Var}(X)$.
1c. Find $E(Y)$.
1d. Find $\text{Var}(Y)$.

2. (Same problem setup as (1).) She pays $10 at the start, to play the game. Suppose Matilda wins $1.25 for each die value unequal to “1”. (She doesn’t win anything for the “1” occurring at the end.) What is Matilda’s expected net gain or net loss from such a game?

3. Let $X$ be a geometric random variable with $E(X) = 1/p$. Let $a$ and $b$ be fixed positive integers with $a < b$. Find $P(a \leq X \leq b)$.

4. Let $X$ be a geometric random variable with $E(X) = 1/p$. Let $a$ and $b$ be fixed positive integers with $a < b$.
4a. Find the probability that $X > b$, given $X > a$, i.e., find $P(X > b \mid X > a)$.
4b. Find the probability that $X \leq b$, given $X > a$, i.e., find $P(X \leq b \mid X > a)$.
4c. Find the probability that $X = b$, given $X > a$, i.e., find $P(X = b \mid X > a)$.

5. Suppose, on each round of a game, Alice rolls a 6-sided die, and Bob rolls a 4-sided die. They keep going until the first round on which they both (simultaneously) get values of 1, and then they stop. Let $X$ denote the number of rolls until (and including) that first round on which they both (simultaneously) get 1.
5a. Find $E(X)$.
5b. Find $\text{Var}(X)$.

6. Suppose Alice rolls a (6-sided) dice until she gets her first occurrence of “1” and then she stops. Let $X$ denote the number of rolls until (and including) that first occurrence of “1.” Suppose Bob flips a fair coin until he gets his first occurrence of “heads” and then he stops. Let $Y$ denote the number of flips until (and including) that first occurrence of “heads.” Find $P(X \geq Y)$. 