STAT/MA 41600
In-Class Problem Set #16: September 26, 2014

1. Matilda rolls a die until the first occurrence of "1," and then she stops. Let X denote the number of rolls until (and including) that first occurrence of "1"; let Y denote the number of rolls that occur strictly before (but not including) that "1." So we always have $Y = X - 1$.
1a. Find $\mathbb{E}(X)$.
1b. Find $\text{Var}\,(X)$.
1c. Find $\mathbb{E}(Y)$.
1d. Find $\text{Var}\,(Y)$.

2. (Same problem setup as **(1)**.) She pays \$10 at the start, to play the game. Suppose Matilda wins \$1.25 for each die value unequal to "1". (She doesn't win anything for the "1" occurring at the end.) What is Matilda's expected net gain or net loss from such a game?

3. Let X be a geometric random variable with $\mathbb{E}(X) = 1/p$. Let a and b be fixed positive integers with $a < b$. Find $P(a \leq X \leq b)$.

4. Let X be a geometric random variable with $\mathbb{E}(X) = 1/p$. Let a and b be fixed positive integers with $a < b$.
4a. Find the probability that $X > b$, given $X > a$, i.e., find $P(X > b \mid X > a)$.
4b. Find the probability that $X \leq b$, given $X > a$, i.e., find $P(X \leq b \mid X > a)$.
4c. Find the probability that $X = b$, given $X > a$, i.e., find $P(X = b \mid X > a)$.

5. Suppose, on each round of a game, Alice rolls a 6-sided die, and Bob rolls a 4-sided die. They keep going until the first round on which they both (simultaneously) get values of 1, and then they stop. Let X denote the number of rolls until (and including) that first round on which they both (simultaneously) get 1.
5a. Find $\mathbb{E}(X)$.
5b. Find $\text{Var}\,(X)$.

6. Suppose Alice rolls a (6-sided) dice until she gets her first occurrence of "1" and then she stops. Let X denote the number of rolls until (and including) that first occurrence of "1." Suppose Bob flips a fair coin until he gets his first occurrence of "heads" and then he stops. Let Y denote the number of flips until (and including) that first occurrence of "heads." Find $P(X \geq Y)$.