1. Matilda rolls a die until the eighth occurrence of “1,” and then she stops. Let \(X\) denote the number of rolls until (and including) that eighth occurrence of “1”; let \(Y\) denote the number of rolls not equal to “1” that occur strictly before (but not including) that eighth “1.” So we always have \(Y = X - 8\).

1a. Find \(E(X)\).
1b. Find \(Var(X)\).
1c. Find \(E(Y)\).
1d. Find \(Var(Y)\).

2. (Same problem setup as (1).) She pays $54 at the start, to play the game. Suppose Matilda wins $1.25 for each die value unequal to “1”. (She doesn’t win anything for any of the “1”s, throughout the game.) What is Matilda’s expected net gain or net loss from such a game?

3. Let \(X\) be a negative binomial random variable with parameters \(r\) and \(p\). Give a (relatively simple) expression for \(P(X \leq r + 2)\).

4. Let \(X_1, X_2, X_3, X_4, X_5\) be independent Geometric random variables, with \(E(X_j) = 3\) for each \(j\). Define \(X = X_1 + X_2 + X_3 + X_4 + X_5\).

4a. Find \(P(X > 7 \mid X > 5)\).
4b. Find \(P(X \leq 7 \mid X > 5)\).
4c. Find \(P(X = 7 \mid X > 5)\).

5. At a certain college, 40% of the students live in a residence hall (on-campus), and the other 60% of the students live off-campus. Suppose that Audrey independently selects and interviews people, and she stops after she has found 6 students who live in a residence hall. Let \(X\) denote the number of interviews she conducts altogether.

5a. Find \(E(X)\).
5b. Find \(Var(X)\).
5c. Find \(P(X \geq 9)\).

6a. Suppose Alice rolls a 6-sided dice until she gets her eighth occurrence of “1” and then she stops. Let \(X\) denote the number of rolls until (and including) that eighth occurrence of “1.” Find \(E(X)\) and \(Var(X)\).

6b. Suppose Bob rolls a 6-sided dice until he gets his first occurrence of “1” and then he stops. Let \(Y\) denote the number of rolls until (and including) that first occurrence of “1.” Let \(Z = 8Y\). Find \(E(Z)\) and \(Var(Z)\).

6c. Suppose Christine rolls an 8-sided dice until she gets her sixth occurrence of “1” and then she stops. Let \(U\) denote the number of rolls until (and including) that sixth occurrence of “1.” Find \(E(U)\) and \(Var(U)\).

6d. Suppose Daphne rolls an 8-sided dice until she gets her first occurrence of “1” and then she stops. Let \(V\) denote the number of rolls until (and including) that first occurrence of “1.” Let \(W = 6V\). Find \(E(W)\) and \(Var(W)\).