

STAT/MA 41600
 In-Class Problem Set #17: September 29, 2014
 Solutions by Mark Daniel Ward

1. Since X is a Negative Binomial random variable with $r = 8$ and $p = 1/6$, then $\mathbb{E}(X) = r/p = 48$ and $\text{Var}(X) = rq/p^2 = (8)(5/6)/(1/6)^2 = 240$. Since $Y = X - 8$ then $\mathbb{E}(Y) = \mathbb{E}(X - 8) = \mathbb{E}(X) - 8 = 48 - 8 = 40$ and $\text{Var}(Y) = \text{Var}(X - 1) = \text{Var}(X) = 240$.

2. Matilda wins $1.25Y - 54$, so her expected net winnings are $\mathbb{E}(1.25Y - 54) = 1.25\mathbb{E}(Y) - 54 = 1.25(40) - 54 = -4$. In other words, her expected net loss is 4.

3. We know $X \geq r$, so we have $P(X \leq r + 2) = \sum_{x=r}^{r+2} \binom{x-1}{r-1} q^{x-r} p^r = p^r (1 + rq + \frac{r(r+1)}{2} q^2)$.

4a. Since X is Negative Binomial with $r = 5$ and $p = 1/3$, then $P(X > 7 | X > 5) = \frac{P(X > 7 \& X > 5)}{P(X > 5)} = \frac{P(X > 7)}{1 - P(X \leq 5)} = \frac{1 - P(X = 7) - P(X = 6) - P(X = 5)}{1 - P(X = 5)} = \frac{1 - \binom{6}{4} q^2 p^5 - \binom{5}{4} q p^5 - \binom{4}{4} p^5}{1 - \binom{4}{4} p^5} = \frac{116}{121}$.

4b. We have $P(X \leq 7 | X > 5) = 1 - (X > 7 | X > 5) = 5/121$.

4c. We have $P(X = 7 | X > 5) = \frac{P(X = 7 \& X > 5)}{P(X > 5)} = \frac{P(X = 7)}{1 - P(X = 5)} = \frac{P(X = 7)}{1 - \binom{4}{4} p^5} = \frac{\binom{6}{4} q^2 p^5}{1 - \binom{4}{4} p^5} = 10/363$.

5. Since X is a Negative Binomial random variable with $r = 6$ and $p = 2/5$ then $\mathbb{E}(X) = r/p = 15$ and $\text{Var}(X) = rq/p^2 = 45/2$. Also $P(X \geq 9) = 1 - P(X < 9) = 1 - P(X = 8) - P(X = 7) - P(X = 6) = 1 - \binom{7}{5} q^2 p^6 - \binom{6}{5} q p^6 - \binom{5}{5} p^6 = 371169/390625 = 0.95019264$.

6a. Since X is Negative Binomial with $r = 8$ and $p = 1/6$, then $\mathbb{E}(X) = r/p = 48$ and $\text{Var}(X) = rq/p^2 = 240$.

6b. Since Y is Geometric with $p = 1/6$, then $\mathbb{E}(Y) = 1/p = 6$ and $\text{Var}(Y) = q/p^2 = 30$, so $\mathbb{E}(Z) = \mathbb{E}(8Y) = 8\mathbb{E}(Y) = 8(6) = 48$ and $\text{Var}(Z) = \text{Var}(8Y) = 8^2 \text{Var}(Y) = 8^2(30) = 1920$.

6c. Since U is Negative Binomial with $r = 6$ and $p = 1/8$, then $\mathbb{E}(U) = r/p = 48$ and $\text{Var}(U) = rq/p^2 = 336$.

6d. Since V is Geometric with $p = 1/8$, then $\mathbb{E}(V) = 1/p = 8$ and $\text{Var}(V) = q/p^2 = 56$, so $\mathbb{E}(W) = \mathbb{E}(6V) = 6\mathbb{E}(V) = 6(8) = 48$ and $\text{Var}(W) = \text{Var}(6V) = 6^2 \text{Var}(V) = 6^2(56) = 2016$.