

STAT/MA 41600  
 In-Class Problem Set #17: September 29, 2014  
 Solutions by Mark Daniel Ward

**1.** Since  $X$  is a Negative Binomial random variable with  $r = 8$  and  $p = 1/6$ , then  $\mathbb{E}(X) = r/p = 48$  and  $\text{Var}(X) = rq/p^2 = (8)(5/6)/(1/6)^2 = 240$ . Since  $Y = X - 8$  then  $\mathbb{E}(Y) = \mathbb{E}(X - 8) = \mathbb{E}(X) - 8 = 48 - 8 = 40$  and  $\text{Var}(Y) = \text{Var}(X - 8) = \text{Var}(X) = 240$ .

**2.** Matilda wins  $1.25Y - 54$ , so her expected net winnings are  $\mathbb{E}(1.25Y - 54) = 1.25\mathbb{E}(Y) - 54 = 1.25(40) - 54 = -4$ . In other words, her expected net loss is 4.

**3.** We know  $X \geq r$ , so we have  $P(X \leq r + 2) = \sum_{x=r}^{r+2} \binom{x-1}{r-1} q^{x-r} p^r = p^r(1 + rq + \frac{r(r+1)}{2}q^2)$ .

**4a.** Since  $X$  is Negative Binomial with  $r = 5$  and  $p = 1/3$ , then  $P(X > 7 | X > 5) = \frac{P(X > 7 \& X > 5)}{P(X > 5)} = \frac{P(X > 7)}{1 - P(X \leq 5)} = \frac{1 - P(X=7) - P(X=6) - P(X=5)}{1 - P(X=5)} = \frac{1 - \binom{6}{4}q^2p^5 - \binom{5}{4}qp^5 - \binom{4}{4}p^5}{1 - \binom{4}{4}p^5} = \frac{116}{121}$ .

**4b.** We have  $P(X \leq 7 | X > 5) = 1 - (X > 7 | X > 5) = 5/121$ .

**4c.** We have  $P(X = 7 | X > 5) = \frac{P(X=7 \& X > 5)}{P(X > 5)} = \frac{P(X=7)}{P(X > 5)} = \frac{P(X=7)}{1 - P(X=5)} = \frac{\binom{6}{4}q^2p^5}{1 - \binom{4}{4}p^5} = 10/363$ .

**5.** Since  $X$  is a Negative Binomial random variable with  $r = 6$  and  $p = 2/5$  then  $\mathbb{E}(X) = r/p = 15$  and  $\text{Var}(X) = rq/p^2 = 45/2$ . Also  $P(X \geq 9) = 1 - P(X < 9) = 1 - P(X = 8) - P(X = 7) - P(X = 6) = 1 - \binom{7}{5}q^2p^6 - \binom{6}{5}qp^6 - \binom{5}{5}p^6 = 371169/390625 = 0.95019264$ .

**6a.** Since  $X$  is Negative Binomial with  $r = 8$  and  $p = 1/6$ , then  $\mathbb{E}(X) = r/p = 48$  and  $\text{Var}(X) = rq/p^2 = 240$ .

**6b.** Since  $Y$  is Geometric with  $p = 1/6$ , then  $\mathbb{E}(Y) = 1/p = 6$  and  $\text{Var}(Y) = q/p^2 = 30$ , so  $\mathbb{E}(Z) = \mathbb{E}(8Y) = 8\mathbb{E}(Y) = 8(6) = 48$  and  $\text{Var}(Z) = \text{Var}(8Y) = 8^2 \text{Var}(Y) = 8^2(30) = 1920$ .

**6c.** Since  $U$  is Negative Binomial with  $r = 6$  and  $p = 1/8$ , then  $\mathbb{E}(U) = r/p = 48$  and  $\text{Var}(U) = rq/p^2 = 336$ .

**6d.** Since  $V$  is Geometric with  $p = 1/8$ , then  $\mathbb{E}(V) = 1/p = 8$  and  $\text{Var}(V) = q/p^2 = 56$ , so  $\mathbb{E}(W) = \mathbb{E}(6V) = 6\mathbb{E}(V) = 6(8) = 48$  and  $\text{Var}(W) = \text{Var}(6V) = 6^2 \text{Var}(V) = 6^2(56) = 2016$ .