STAT/MA 41600
In-Class Problem Set #18: October 1, 2014

1. Catherine watches raindrops hit the window. The number of raindrops that fall in a fixed period of time is Poisson with an average of 6 per minute.
1a. What is the probability that exactly 5 raindrops fall during the next one minute?
1b. What is the probability that no raindrops fall during the next one minute?
1c. What is the probability that 4 or more raindrops fall during the next one minute?

2. (Same problem setup as **(1)**.)
2a. What is the probability that exactly 1 raindrop falls during the next 15 seconds?
2b. What is the probability that no raindrops fall during the next 15 seconds?
2c. What is the probability that 3 or more raindrops fall during the next 15 seconds?

3. Let X be a Poisson random variable with $\mathbb{E}(X) = \lambda$. Give a (relatively simple) expression for $P(X \geq 4)$.

4. Dr. Ward estimates that, among 10,000 students on campus, the students choose independently whether to visit the MATH library on a given day. Each student decides to visit with probability 1/1000, or to avoid the MATH library with probability 999/1000. Let X denote the number of people who visit the MATH library on a given day.
4a. What is the mass of X?
4b. What is $\mathbb{E}(X)$?
4c. What is $\mathrm{Var}\,(X)$?
4d. Estimate the probability that 9 people visit the MATH library on a given day.

5. Dr. Ward estimates that, among 10,000 students on campus, the students choose independently whether to go to Purdue Salvage on a given day. Each student decides to go to Purdue Salvage probability 3/10000, or to avoid going to Purdue Salvage with probability 9997/10000. Let X denote the number of people who go to Purdue Salvage on a given day.
5a. Estimate $P(X \geq 5)$.

Now suppose that, among the 10,000 students on campus, the students choose independently whether to go to the US postal office on a given day. Each student decides to go to the US postal office with probability 5/10000, or to avoid going to the US postal office with probability 9995/10000. Let Y denote the number of people who go to the US postal office on a given day. Also assume that the Purdue Salvage and US postal office choices are independent, so X and Y are independent.
5b. Estimate $P(8 \leq X + Y \leq 9)$. [Hint: The distributions of X and Y are each approximately Poisson, and they are independent, so $X + Y$ is approximately Poisson too.]

6. If X is a Poisson random variable with parameter λ and Y is a Geometric random variable with $\mathbb{E}(Y) = 1/p$, and if X and Y are independent, find $P(Y > X)$. Hint: $P(Y > X) = \sum_{x=0}^{\infty} \sum_{y=x+1}^{\infty} p_{X,Y}(x, y)$.