STAT/MA 41600
In-Class Problem Set #19: October 3, 2014

1. A professor estimates that, among 15 students in a seminar, 5 of them enjoyed the seminar that day, and the other 10 did not enjoy it. He interviews 3 of the people in the class, selected at random and without replacement, and he lets X denote the number of people (among these 3) who enjoyed the seminar.
1a. What is the mass of X?
1b. What is $\mathbb{E}(X)$?
1c. What is $\mathrm{Var}\,(X)$?

2. When rolling a die, a "high value" is a 5 or 6. Roll 10 dice. Suppose that 8 of them are "high values" and 2 of them are not "high values." If we examine four dice in this collection (without replacement), chosen at random, find the probability that two or fewer (of these four chosen dice) are "high values".

3. Six rocks are sitting in a row. We paint them, using up to three colors (say, R's, W's, and B's). Suppose all of the $3^6 = 729$ outcomes are equally likely.
3a. If 3 of the 6 rocks are chosen—without replacement—what is the probability that 2 or more of 3 chosen rocks are blue?
3b. If (instead, at the start) 3 of the 6 rocks are chosen—*with replacement*—what is the probability that 2 or more of 3 chosen rocks are blue?

4. Suppose that X is Binomial with parameters $n = 3$ and $p = 1/3$. Suppose that Y is a Hypergeometric random variable with parameters $N = 10$, $M = 4$, and $n = 3$. Also suppose that X and Y are independent. Find $P(X \geq Y)$.

5. Suppose Alice chooses (without replacement) 3 toy stuffed animals from a box that contains 8 stuffed turtles and 2 stuffed mice. Let X denote the number of stuffed turtles that she gets. Suppose that Bob chooses (but *with replacement* in between selections) 3 real animals from a zoo exhibit that has 8 turtles and 2 mice. Let Y denote the number of turtles that he gets. It is safe to assume that Alice and Bob's methods of selection are independent, so X and Y are independent. Find $P(X = Y)$.

6. A chemistry professor chooses 4 chemicals (without replacement) from the cabinet. There are 12 chemicals present: 9 of them are toxic, and the other 3 are non-toxic. Let X_j indicate whether the jth choice is toxic, for $j = 1, \ldots, 4$. In other words, $X_j = 1$ if the jth chemical chosen is toxic, or $X_j = 0$ otherwise. Write $X = X_1 + X_2 + X_3 + X_4$.
6a. Using the fact that $X^2 = (X_1+X_2+X_3+X_4)^2$ find $\mathbb{E}(X^2)$ by expanding the 16-term sum.
6b. Find the mass of X.
6c. Use your answer from **(6b)** to directly verify the answer of **(6a)** by a technique that uses that mass of X.
6d. Find $\mathbb{E}(X)$.
6e. Find $\mathrm{Var}\,(X)$.