

STAT/MA 41600  
In-Class Problem Set #20/#22: October 6, 2014  
(there is no Problem Set #21)

**1.** A standard deck of 52 cards is shuffled. A person draws them until the first “10” appears, and then stops. Let  $X$  denote the number of cards that appear before the first “10” appears (not including the first “10” itself). [[Hint: For each of the 48 non-10’s, check whether the card appears before all the 10’s or not. Write  $X = X_1 + \cdots + X_{48}$  where  $X_j = 1$  if the  $j$ th card of the 48 non-10’s appears before all the 10’s, and  $X_j = 0$  otherwise.]]

**1a.** What is  $\mathbb{E}(X)$ ?

**1b.** What is  $\text{Var}(X)$ ?

**2.** There are 5 math books, 7 biology books, and 10 history books, all sitting on a shelf. Suppose we randomly take 6 of the books, with all selections equally likely.

**2a.** What is the probability that we get 2 books from each subject?

**2b.** What is the probability that all 6 books are chosen from the same subject?

**3.** There are 7 Statistics students and 7 Mathematics students. They are randomly split into 7 groups, with 2 students per group (all such arrangements are equally likely). Let  $X$  denote the number of groups consisting of 1 Statistics student and 1 Mathematics student. Find  $\mathbb{E}(X)$ .

**4a.** In a bag containing 7 candy corns and 9 gumdrops, we choose exactly 3 pieces altogether. Let  $X$  denote the number of candy corns chosen, when the pieces are chosen with replacement. Find  $\text{Var}(X)$ .

**4b.** In a bag containing 7 candy corns and 9 gumdrops, we choose exactly 3 pieces altogether. Let  $Y$  denote the number of candy corns chosen, when the pieces are chosen *without replacement*. Find  $\text{Var}(Y)$ .

**5.** Alice, Bob, Claudine, and David independently each pick a digit from 0 to 9.

**5a.** Find the probability that they all pick a unique digit, i.e., that they have no overlap among their choices.

**5b.** Find the probability that they all pick a unique digit, and among these unique digits, from smallest-to-largest, the digits belong to Alice (smallest), Bob (second-smallest), Claudine (second-largest), and David (largest).

**6.** Suppose that  $X$  is equally likely to be any of the values in the list 5, 7, 9, 11, 13, 15, 17, 19.

**6a.** Find  $\mathbb{E}(X)$ .

**6b.** Find  $\text{Var}(X)$ .

[[Hint: Write  $X = 2Y + 3$  where  $Y$  is a discrete uniform random variable with possible values 1, 2,  $\dots$ , 8.]]