

STAT/MA 41600
In-Class Problem Set #24: October 15, 2014
(there is no Problem Set #23)

1. Suppose that X has probability density function $f_X(x) = \frac{3}{5}e^{-(3/5)x}$ for $x > 0$, and $f_X(x) = 0$ otherwise.

1a. What is an exact expression for $P(2 \leq X \leq 7)$?

1b. Compute the expression in part **1a** on your calculator.

1c. If a and b are positive constants with $a < b$, find an expression for $P(a \leq X \leq b)$.

2. Suppose that X has density $f_X(x) = kx(1-x)^3$ for $0 \leq x \leq 1$, and $f_X(x) = 0$ otherwise.

2a. Find the value of k that makes this a valid probability density function.

2b. Find $P(X \geq 1/2)$.

3. Consider a random variable X with constant density on the interval $[-20, 20]$.

3a. Compute $P(X \geq 5)$.

3b. Compute $P(|X| \leq 3)$.

4. A certain random variable X has CDF

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0 \\ x^2(2-x)^2 & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

4a. Compute $P(X \leq 1/4)$.

4b. Compute $P(3/4 \leq X)$.

4c. Compute $P(3/8 \leq X \leq 5/8)$.

(Decimal numbers are OK if you don't want to write out the exact fractional values.)

5. If X has density function $f_X(x) = (\frac{1}{18})\sqrt{x+3}$ for $-3 \leq x \leq 6$ and $f_X(x) = 0$ otherwise, find the probability that X is positive. Give a decimal value for this probability too, i.e., compute the probability on your calculator after you find the exact probability.

6a. For the CDF given in question **4**, find the probability density function $f_X(x)$.

6b. For the density function in question **2**, find the cumulative distribution function $F_X(x)$.