

STAT/MA 41600
In-Class Problem Set #24: October 15, 2014
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1a. The exact expression is $P(2 \leq X \leq 7) = \int_2^7 \frac{3}{5} e^{-(3/5)x} dx = e^{-6/5} - e^{-21/5}$.

1b. On our calculator, we get $P(2 \leq X \leq 7) = \int_2^7 \frac{3}{5} e^{-(3/5)x} dx = e^{-6/5} - e^{-21/5} = 0.2862$.

1c. For any positive constants a, b with $a < b$, we have $P(a \leq X \leq b) = \int_a^b \frac{3}{5} e^{-(3/5)x} dx = e^{-(3/5)a} - e^{-(3/5)b}$.

2a. We compute $\int_0^1 x(1-x)^3 dx = 1/20$, so multiplying throughout by 20, we see that $\int_0^1 20x(1-x)^3 dx = 1$. Thus k must equal 20.

2b. We have $P(X \geq 1/2) = \int_{1/2}^1 20x(1-x)^3 dx = 3/16$.

3a. The constant density must be $f_X(x) = 1/40$ for $-20 \leq x \leq 20$, and $f_X(x) = 0$ otherwise. So $P(X \geq 5) = \int_5^{20} 1/40 = 3/8$. Alternatively, *since the density is constant*, we can compute with lengths, i.e., $P(X \geq 5) = (\text{length of } [5, 20]) / (\text{length of } [-20, 20]) = 15/40 = 3/8$.

3b. We have $P(|X| \leq 3) = P(-3 \leq X \leq 3) = \int_{-3}^3 1/40 = 3/20$. Alternatively, *since the density is constant*, we can compute with lengths, i.e., $P(|X| \leq 3) = P(-3 \leq X \leq 3) = (\text{length of } [-3, 3]) / (\text{length of } [-20, 20]) = 6/40 = 3/20$.

4a. We have $P(X \leq 1/4) = F_X(1/4) = 49/256$.

4b. We have $P(3/4 \leq X) = 1 - P(X \leq 3/4) = 1 - F_X(3/4) = 1 - 225/256 = 31/256$.

4c. We have $P(3/8 \leq X \leq 5/8) = P(X \leq 5/8) - P(X \leq 3/8) = F_X(5/8) - F_X(3/8) = 3025/4096 - 1521/4096 = 47/128$.

5. The probability is $P(X > 0) = \int_0^6 (\frac{1}{18}) \sqrt{x+3} dx = 1 - \frac{\sqrt{3}}{9} = 0.8075$.

6a. The density function is $f_X(x) = F'(x)$, so $f_X(x) = 4x^3 - 12x^2 + 8x$ for $0 \leq x \leq 1$, and $f_X(x) = 0$ otherwise.

6b. We have $F_X(x) = 0$ for $x < 0$ and $F_X(x) = 1$ for $x > 1$. For $0 \leq a \leq 1$, we have $F_X(a) = \int_0^a 20x(1-x)^3 dx = -4a^5 + 15a^4 - 20a^3 + 10a^2$. Thus $F_X(x) = -4x^5 + 15x^4 - 20x^3 + 10x^2$ for $0 \leq x \leq 1$.