

STAT/MA 41600  
In-Class Problem Set #26: October 20, 2014

**1.** Consider a pair of random variables  $X, Y$  with constant joint density on the quadrilateral with vertices located at the points  $(0, 0), (3, 0), (5, 2), (0, 2)$ .

- 1a.** Are  $X$  and  $Y$  independent? Why or why not?
- 1b.** Find the density  $f_X(x)$  of  $X$ .
- 1c.** Find the density  $f_Y(y)$  of  $Y$ .

**2.** Suppose that  $X$  and  $Y$  have a constant joint density on the triangle with vertices  $(0, 0), (3, 0), (0, 3)$ .

- 2a.** Are  $X$  and  $Y$  independent? Why or why not?
- 2b.** Find the density  $f_X(x)$  of  $X$ .
- 2c.** Find the density  $f_Y(y)$  of  $Y$ .

**3.** Suppose  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = 6e^{-3x-2y}$$

for  $x > 0$  and  $y > 0$ ; and  $f_{X,Y}(x, y) = 0$  otherwise.

- 3a.** Are  $X$  and  $Y$  independent? Why or why not?
- 3b.** Find the density  $f_X(x)$  of  $X$ .
- 3c.** Find the density  $f_Y(y)$  of  $Y$ .

**4.** Suppose  $X, Y$  has joint density

$$f_{X,Y}(x, y) = \begin{cases} \frac{9}{256}x^2y^2 & \text{if } -2 < x < 2 \text{ and } -2 < y < 2 \\ 0 & \text{otherwise} \end{cases}$$

- 4a.** Are  $X$  and  $Y$  independent? Why or why not?
- 4b.** Find the density  $f_X(x)$  of  $X$ .
- 4c.** Find the density  $f_Y(y)$  of  $Y$ .

**5.** Suppose that  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = \begin{cases} \frac{1}{36}(4-x)(3-y) & \text{if } 0 < x < 4 \text{ and } 0 < y < 3 \\ 0 & \text{otherwise} \end{cases}$$

- 5a.** Find  $\mathbb{E}(XY)$ .
- 5b.** Find  $\mathbb{E}(X - Y)$ .

**6.** For the random variables given in question **3**, let  $W = \min(X, Y)$ .

- 6a.** Find the density  $f_W(w)$  of  $W$ .
- 6b.** Find  $\mathbb{E}(W)$ .

This problem set has a preview of how to compute expected values for continuous random variables. Expected values for continuous random variables are computed in the same way as for discrete random variables, except we integrate instead of summing, and we use the density instead of the mass function. Otherwise, the method is the same, and we will explore this idea further in upcoming problem sets.