1. Consider a pair of random variables $X, Y$ with constant joint density on the quadrilateral with vertices located at the points $(0,0), (3,0), (5,2), (0,2)$.

1a. For $0 \leq y \leq 2$, find the conditional density $f_{X|Y}(x \mid y)$ of $X$, given $Y = y$.

1b. Find the conditional probability that $X \leq 3$, given $Y = 1$. I.e., find $P(X \leq 3 \mid Y = 1)$.

1c. Find the conditional probability that $X \leq 3$, given $Y \leq 1$. I.e., find $P(X \leq 3 \mid Y \leq 1)$.

2. Same setup as in question #1.

2a. For $0 \leq x \leq 3$, find the conditional density $f_{Y|X}(y \mid x)$ of $Y$, given $X = x$.

2b. For $3 \leq x \leq 5$, find the conditional density $f_{Y|X}(y \mid x)$ of $Y$, given $X = x$.

2c. Find the conditional probability that $Y \leq 1.5$, given $X = 2$. I.e., find $P(Y \leq 1.5 \mid X = 2)$.

3. Suppose $X$ and $Y$ have joint probability density function

$$f_{X,Y}(x,y) = 10e^{-3x-2y}$$

for $0 < x < y$; and $f_{X,Y}(x,y) = 0$ otherwise. (Note that $X$ and $Y$ are not independent, since we are insisting that $X < Y$ in this setup!)

3a. For $x > 0$, find the conditional density $f_{Y|X}(y \mid x)$ of $Y$, given $X = x$.

3b. Find the conditional probability that $Y > 3$, given $X = 2$, i.e., find $P(Y > 3 \mid X = 2)$.

3c. Find the conditional probability that $Y > 3$, given $X > 2$, i.e., find $P(Y > 3 \mid X > 2)$.

4. Same setup as in question #3.

4a. For $y > 0$, find the conditional density $f_{X|Y}(x \mid y)$ of $X$, given $Y = y$.

4b. Find the conditional probability that $X > 2$, given $Y = 3$, i.e., find $P(X > 2 \mid Y = 3)$.

4c. Find the conditional probability that $X > 2$, given $Y > 3$, i.e., find $P(X > 2 \mid Y > 3)$.

5. Suppose $X$ and $Y$ have joint probability density function

$$f_{X,Y}(x,y) = \frac{1}{2}(2-x)(2-y)$$

for $0 < x < y < 2$; and $f_{X,Y}(x,y) = 0$ otherwise. (Note that $X$ and $Y$ are not independent, since we are again insisting that $X < Y$ in this setup!)

5a. For $0 < x < 2$, find the conditional density $f_{Y|X}(y \mid x)$ of $Y$, given $X = x$.

5b. Find the conditional probability that $Y > 1$, given $X = 1/2$, i.e., find $P(Y > 1 \mid X = 1/2)$.

5c. Find the conditional probability that $Y > 1$, given $X > 1/2$, i.e., find $P(Y > 1 \mid X > 1/2)$.

6. Same setup as in question #5.

6a. For $0 < y < 2$, find the conditional density $f_{X|Y}(x \mid y)$ of $X$, given $Y = y$.

6b. Find the conditional probability that $X > 1/2$, given $Y = 1$, i.e., find $P(X > 1/2 \mid Y = 1)$.

6c. Find the conditional probability that $X > 1/2$, given $Y > 1$, i.e., find $P(X > 1/2 \mid Y > 1)$. 