

STAT/MA 41600
In-Class Problem Set #27: October 22, 2014

1. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices located at the points $(0, 0), (3, 0), (5, 2), (0, 2)$.
 - 1a. For $0 \leq y \leq 2$, find the conditional density $f_{X|Y}(x | y)$ of X , given $Y = y$.
 - 1b. Find the conditional probability that $X \leq 3$, given $Y = 1$. I.e., find $P(X \leq 3 | Y = 1)$.
 - 1c. Find the conditional probability that $X \leq 3$, given $Y \leq 1$. I.e., find $P(X \leq 3 | Y \leq 1)$.
2. Same setup as in question #1.
 - 2a. For $0 \leq x \leq 3$, find the conditional density $f_{Y|X}(y | x)$ of Y , given $X = x$.
 - 2b. For $3 \leq x \leq 5$, find the conditional density $f_{Y|X}(y | x)$ of Y , given $X = x$.
 - 2c. Find the conditional probability that $Y \leq 1.5$, given $X = 2$. I.e., find $P(Y \leq 1.5 | X = 2)$.
3. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 10e^{-3x-2y}$$

for $0 < x < y$; and $f_{X,Y}(x, y) = 0$ otherwise. (Note that X and Y are not independent, since we are insisting that $X < Y$ in this setup!)

- 3a. For $x > 0$, find the conditional density $f_{Y|X}(y | x)$ of Y , given $X = x$.
 - 3b. Find the conditional probability that $Y > 3$, given $X = 2$, i.e., find $P(Y > 3 | X = 2)$.
 - 3c. Find the conditional probability that $Y > 3$, given $X > 2$, i.e., find $P(Y > 3 | X > 2)$.
4. Same setup as in question #3.
 - 4a. For $y > 0$, find the conditional density $f_{X|Y}(x | y)$ of X , given $Y = y$.
 - 4b. Find the conditional probability that $X > 2$, given $Y = 3$, i.e., find $P(X > 2 | Y = 3)$.
 - 4c. Find the conditional probability that $X > 2$, given $Y > 3$, i.e., find $P(X > 2 | Y > 3)$.

5. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = \frac{1}{2}(2-x)(2-y)$$

for $0 < x < y < 2$; and $f_{X,Y}(x, y) = 0$ otherwise. (Note that X and Y are not independent, since we are again insisting that $X < Y$ in this setup!)

- 5a. For $0 < x < 2$, find the conditional density $f_{Y|X}(y | x)$ of Y , given $X = x$.
 - 5b. Find the conditional probability that $Y > 1$, given $X = 1/2$, i.e., find $P(Y > 1 | X = 1/2)$.
 - 5c. Find the conditional probability that $Y > 1$, given $X > 1/2$, i.e., find $P(Y > 1 | X > 1/2)$.
6. Same setup as in question #5.
 - 6a. For $0 < y < 2$, find the conditional density $f_{X|Y}(x | y)$ of X , given $Y = y$.
 - 6b. Find the conditional probability that $X > 1/2$, given $Y = 1$, i.e., find $P(X > 1/2 | Y = 1)$.
 - 6c. Find the conditional probability that $X > 1/2$, given $Y > 1$, i.e., find $P(X > 1/2 | Y > 1)$.