

STAT/MA 41600
In-Class Problem Set #28: October 24, 2014
Solutions by Mark Daniel Ward

1. For $0 \leq x \leq 3$, we have $f_X(x) = \int_0^2 f_{X,Y}(x, y) dy = \int_0^2 1/8 dy = 1/4$. For $3 \leq x \leq 5$, we have $f_X(x) = \int_{x-3}^2 f_{X,Y}(x, y) dy = \int_{x-3}^2 1/8 dy = (5-x)/8$. Thus $\mathbb{E}(X) = \int_0^5 x f_X(x) dx = \int_0^3 (x)(1/4) dx + \int_3^5 (x)(5-x)/8 dx = 49/24$.

2. For $0 \leq y \leq 2$, we have $f_Y(y) = \int_0^{y+3} f_{X,Y}(x, y) dx = \int_0^{y+3} 1/8 dx = (y+3)/8$. Thus $\mathbb{E}(Y) = \int_0^2 y f_Y(y) dy = \int_0^2 (y)(y+3)/8 dy = 13/12$.

3. For $0 < x$, we have $f_X(x) = \int_x^\infty f_{X,Y}(x, y) dy = \int_x^\infty 10e^{-3x-2y} dy = 5e^{-5x}$. Thus $\mathbb{E}(X) = \int_0^\infty x f_X(x) dx = \int_0^\infty (x)(5e^{-5x}) dx = 1/5$.

4. For $0 < y$, we have $f_Y(y) = \int_0^y f_{X,Y}(x, y) dx = \int_0^y 10e^{-3x-2y} dx = (10/3)(e^{-2y} - e^{-5y})$. Thus $\mathbb{E}(Y) = \int_0^\infty y f_Y(y) dy = \int_0^\infty (y)(10/3)(e^{-2y} - e^{-5y}) dy = 7/10$.

5. For $0 < x < 2$, we have $f_X(x) = \int_x^2 f_{X,Y}(x, y) dy = \int_x^2 \frac{1}{2}(2-x)(2-y) dy = (1/4)(2-x)^3$. Thus $\mathbb{E}(X) = \int_0^2 x f_X(x) dx = \int_0^2 (x)(1/4)(2-x)^3 dx = 2/5$.

6. For $0 < y < 2$, we have $f_Y(y) = \int_0^y f_{X,Y}(x, y) dx = \int_0^y \frac{1}{2}(2-x)(2-y) dx = (1/4)y(2-y)(4-y)$. Thus $\mathbb{E}(Y) = \int_0^2 y f_Y(y) dy = \int_0^2 (y)(1/4)y(2-y)(4-y) dy = 14/15$.