

STAT/MA 41600
In-Class Problem Set #29: October 27, 2014
Solutions by Mark Daniel Ward

1. We have $\mathbb{E}(XY) = \int_0^2 \int_0^{y+3} (xy)(1/8) dx dy = 19/8$.

2a. We know, from last assignment, $\mathbb{E}(X) = 49/24$. Also $\mathbb{E}(X^2) = \int_0^2 \int_0^{y+3} (x^2)(1/8) dx dy = 17/3$. Thus $\text{Var}(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = 17/3 - (49/24)^2 = 863/576$.

2a. We know, from last assignment, $\mathbb{E}(Y) = 13/12$. Also $\mathbb{E}(Y^2) = \int_0^2 \int_0^{y+3} (y^2)(1/8) dx dy = 3/2$. Thus $\text{Var}(Y) = \mathbb{E}(Y^2) - (\mathbb{E}(Y))^2 = 3/2 - (13/12)^2 = 47/144$.

3. We have $\mathbb{E}(X^2) = \int_0^\infty \int_x^\infty (x^2)(10e^{-3x-2y}) dy dx = 2/25$.

4. We have $\mathbb{E}(Y^2) = \int_0^\infty \int_x^\infty (y^2)(10e^{-3x-2y}) dy dx = 39/50$.

5. We have $\mathbb{E}(XY) = \int_0^2 \int_x^2 (xy)(1/2)(2-x)(2-y) dy dx = 4/9$.

6a. We have $\mathbb{E}((XY)^2) = \int_0^2 \int_x^2 (xy)^2(1/2)(2-x)(2-y) dy dx = 4/9$.

6b. We have $\text{Var}(XY) = \mathbb{E}((XY)^2) - (\mathbb{E}(XY))^2 = 4/9 - (4/9)^2 = 20/81$.