

STAT/MA 41600
In-Class Problem Set #31: October 29, 2014
(there is no Problem Set #30)
Solutions by Mark Daniel Ward

1. We have $\mathbb{E}(\max(X, Y)) = \int_0^{5/2} \int_y^{5-y} (x) \left(\frac{2}{25}\right) dx dy + \int_0^{5/2} \int_x^{5-x} (y) \left(\frac{2}{25}\right) dy dx = \frac{5}{4} + \frac{5}{4} = \frac{5}{2}$.

2ab. We have $f_X(x) = \frac{d}{dx} F_X(x) = \frac{d}{dx} \left(\frac{x-3}{4}\right) = 1/4$ for $3 < x < 7$, and $f_X(x) = 0$ otherwise. So X is a continuous Uniform on $(3, 7)$. Thus $\mathbb{E}(X) = \frac{3+7}{2} = 5$ and $\text{Var}(X) = \frac{(7-3)^2}{12} = 4/3$.

3ab. We have $F_X(x) = P(X \leq x) = P(U \leq x, V \leq x, W \leq x) = P(U \leq x)P(V \leq x)P(W \leq x) = (x/5)^3 = x^3/125$ for $0 \leq x \leq 5$, and $F_X(x) = 0$ for $x < 0$, and $F_X(x) = 1$ for $x > 5$. Thus $f_X(x) = \frac{d}{dx} F_X(x) = 3x^2/125$ for $0 \leq x \leq 5$, and $f_X(x) = 0$ otherwise.

4a. We have $P(U > 7) = \int_7^{10} 1/10 dx = 3/10$.

4b. We have $\text{Var}(U) = (10 - 0)^2/12 = 25/3$, so the standard deviation of U is $\sqrt{25/3}$.

5ab. Since X is uniformly distributed on $[5, 20]$, then $\mathbb{E}(X) = (5 + 20)/2 = 25/2$ and $\text{Var}(X) = (20 - 5)^2/12 = 75/4$. Alternatively, $\mathbb{E}(X) = \mathbb{E}(3Y + 5) = 3\mathbb{E}(Y) + 5 = 3(5/2) + 5$ and $\text{Var}(X) = \text{Var}(3Y + 5) = 3^2 \text{Var}(Y) = 9(5 - 0)^2/12 = 75/4$.

6. Ties of any kind (among the 4 random variables) have probability 0, and each of the 4 random variables are equally likely to be the largest. So U is the largest with probability $1/4$.