1. Let $V$ and $W$ denote (respectively) the times until the next vampire and wereewolf appear at the door for Halloween candy. Suppose that $V, W$ are independent exponential random variables, with $\mathbb{E}(V) = 1/3$ and $\mathbb{E}(W) = 1/5$.

1a. Find the probability the next monster is a vampire, i.e., $P(V < W)$.
1b. Find the probability that both types of monsters have arrived by time $1/10$, e.g., find $P(\max(V, W) \leq 1/10)$.

2. Same setup as #1. Let $X = \min(V, W)$.
2a. Find $P(X > 1/4)$.
2b. Find the median of $X$, i.e., find the “a” so that $P(X \leq a) = 1/2$ and $P(X > a) = 1/2$.

3. Let $X$ and $Y$ denote (respectively) the times until the next witch and wizard apparate. Suppose that $X, Y$ have joint density $f_{X,Y}(x, y) = 14e^{-2x - 7y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.
3a. Find the probability the witch apparates before the wizard, i.e., $P(X < Y)$.
3b. Find the probability at least one of them apparates before time $1/10$, i.e., compute $P(\min(X, Y) \leq 1/10)$.

4. Same setup as #3. Let $U = \max(X, Y)$.
4a. Find the CDF of $U$.
4b. Find the density of $U$.

5. Let $Y$ denote the time until the next black cat screeches in the middle of the night. Suppose that $Y$ is exponential with $\mathbb{E}(Y) = 1/2$. Let $X = 3Y$.
5a. Find $P(X > 1)$.
5b. Find the median of $X$.

6. Same setup as #5.
6a. Find $P(1/2 < X < 3/2)$.
6b. Find $\text{Var}(X)$. 