STAT/MA 41600

In-Class Problem Set #32: October 31, 2014

1. Let V and W denote (respectively) the times until the next vampire and warewolf appear at the door for Halloween candy. Suppose that V, W are independent exponential random variables, with $\mathbb{E}(V) = 1/3$ and $\mathbb{E}(W) = 1/5$.
1a. Find the probability the next monster is a vampire, i.e., $P(V < W)$.
1b. Find the probability that both types of monsters have arrived by time 1/10, e.g., find $P(\max(V, W) \leq 1/10)$.

2. Same setup as **#1**. Let $X = \min(V, W)$.
2a. Find $P(X > 1/4)$.
2b. Find the median of X, i.e., find the "a" so that $P(X \leq a) = 1/2$ and $P(X > a) = 1/2$.

3. Let X and Y denote (respectively) the times until the next witch and wizard apparate. Suppose that X, Y have joint density $f_{X,Y}(x, y) = 14e^{-2x-7y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.
3a. Find the probability the witch apparates before the wizard, i.e., $P(X < Y)$.
3b. Find the probability *at least one* of them apparates before time 1/10, i.e., compute $P(\min(X, Y) \leq 1/10)$.

4. Same setup as **#3**. Let $U = \max(X, Y)$.
4a. Find the CDF of U.
4b. Find the density of U.

5. Let Y denote the time until the next black cat screeches in the middle of the night. Suppose that Y is exponential with $\mathbb{E}(Y) = 1/2$. Let $X = 3Y$.
5a. Find $P(X > 1)$.
5b. Find the median of X.

6. Same setup as **#5**.
6a. Find $P(1/2 < X < 3/2)$.
6b. Find $\mathrm{Var}(X)$.