1a. We have \( P(V < W) = \int_0^\infty \int_v^\infty 15e^{-3v-5w} \, dw \, dv = 3/8 \).

1b. We have \( P(\max(V,W) \leq 1/10) = P(V \leq 1/10 \land W \leq 1/10) = P(V \leq 1/10)P(W \leq 1/10) = (1 - e^{-3(1/10)})(1 - e^{-5(1/10)}) = 0.1020 \).

2a. We have \( P(X > 1/4) = P(\min(V,W) > 1/4) = P(V > 1/4 \land W > 1/4) = P(V > 1/4)P(W > 1/4) = e^{-3(1/4)}e^{-5(1/4)} = e^{-2} = 0.1353 \). Alternatively, since \( X \) is the minimum of independent exponential random variables, then \( X \) is exponential with \( \lambda = 3 + 5 = 8 \), so \( P(X > 1/4) = e^{-(8)(1/4)} = e^{-2} \).

2b. We have \( 1/2 = P(X > a) = e^{-8a} \), so \( \ln(1/2) = -8a \), and thus the median is \( a = \frac{\ln(1/2)}{-8} = (1/8) \ln(2) = 0.0866 \).

3a. We have \( P(X < Y) = \int_0^\infty \int_x^\infty 14e^{-2x-7y} \, dy \, dx = 2/9 \).

3b. Since \( X \) and \( Y \) are independent exponential random variables, then their minimum is an exponential random variable too, with \( \lambda = 2 + 7 = 9 \), so \( P(\min(X,Y) \leq 1/10) = 1 - e^{-9(1/10)} = 0.5934 \).

4a. For \( a > 0 \), we have \( F_U(a) = P(U \leq a) = P(\max(X,Y) \leq a) = P(X \leq a, Y \leq a) = P(X \leq a)P(Y \leq a) = (1 - e^{-2a})(1 - e^{-7a}) \). For \( a \leq 0 \), of course, \( F_U(a) = 0 \).

4b. For \( u > 0 \), we have \( f_U(u) = \frac{d}{du} F_U(u) = \frac{d}{du} ((1 - e^{-2u})(1 - e^{-7u})) = 2e^{-2u} + 7e^{-7u} - 9e^{-9u} \); for \( u \leq 0 \), we have \( f_U(u) = 0 \).

5a. A constant, positive multiple of an exponential random variable is an exponential random variable too. Thus \( X \) is exponential, with \( E(X) = E(3Y) = 3E(Y) = 3/2 \), so \( X \) has parameter \( \lambda = 2/3 \). Thus \( P(X > 1) = \int_1^\infty (2/3)e^{-(2/3)x} \, dx = e^{-2/3} = 0.5134 \).

5b. We have \( 1/2 = P(X > a) = e^{-(2/3)a} \), so \( \ln(1/2) = -(2/3)a \), and thus the median is \( a = \frac{\ln(1/2)}{-(2/3)} = (3/2) \ln(2) = 1.0397 \).

6a. We have \( P(1/2 < X < 3/2) = \int_{1/2}^{3/2} (2/3)e^{-(2/3)x} \, dx = e^{-1/3} - e^{-1} = 0.3487 \).

6b. Since \( X \) is an exponential random variable with \( \lambda = 2/3 \), then \( \text{Var}(X) = 1/\lambda^2 = 9/4 \).