STAT/MA 41600
In-Class Problem Set #32 part 2: November 3, 2014

1. Suppose that V, W are independent exponential random variables, with $\mathbb{E}(V) = 1/3$ and $\mathbb{E}(W) = 1/5$.
1a. Given $W > 1/5$, find the probability $W > 3/5$, i.e., compute $P(W > 3/5 \mid W > 1/5)$.
1b. Given $W < 3/5$, find the probability $W < 1/5$, i.e., compute $P(W < 1/5 \mid W < 3/5)$.

2. Same setup as **#1**.
2a. Let $X = \min(V, W)$. Find $\mathbb{E}(X)$.
2b. Let $Y = \max(V, W)$. Find $\mathbb{E}(Y)$.

3. Suppose that X, Y have joint density $f_{X,Y}(x, y) = 14e^{-2x-7y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise.
3a. Given $X > 1/4$, find the probability $X > 3/4$, i.e., compute $P(X > 3/4 \mid X > 1/4)$.
3b. Given $X < 3/4$, find the probability $X < 1/4$, i.e., compute $P(X < 1/4 \mid X < 3/4)$.

4. Same setup as **#3**.
4a. Let $U = \min(X, Y)$. Find $\mathbb{E}(U)$.
4b. Let $V = \max(X, Y)$. Find $\mathbb{E}(V)$.

5. Let V, W, X, Y be independent exponential random variables with expected values (respectively) 1/3, 1/5, 1/2, 1/7. Let $U = \min(V, W, X, Y)$.
5a. Find $P(U > 1/10)$.
5b. Find $\mathbb{E}(U)$.

6. Same setup as **#5**.
6a. Find $P(U < 1/5 \mid U > 1/20)$.
6b. Find the probability that Y is the smallest of the random variables V, W, X, Y.