

STAT/MA 41600
In-Class Problem Set #33: November 5, 2014

1. Suppose V, W are independent exponential random variables, with $\mathbb{E}(V) = \mathbb{E}(W) = 1/3$. Let $X = V + W$.
 - 1a. What is the standard deviation of X ?
 - 1b. What is the density of X ?
 - 1c. Calculate $P(X \leq 1)$.

2. Same setup as #1.
 - 2a. What is $P(V > W)$? What is $P(V = W)$? What is $P(V < W)$?
 - 2b. Calculate $P(X \geq 1/2)$.

3. Suppose that X, Y have joint density $f_{X,Y}(x, y) = 25e^{-5x-5y}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise. Define $V = X + Y$.
 - 3a. What is the density of V ?
 - 3b. What is the CDF of V ?

4. Same setup as #3.
 - 4a. What is the variance of V ?
 - 4b. Calculate $P(V \leq 1/5)$.

5. Review question: Suppose that X is a Poisson random variable with parameter $\lambda > 0$, and suppose that Y is a Geometric random variable with $\mathbb{E}(Y) = 1/p$. Find $P(Y > X)$. [Hint: You can evaluate a double-sum, and it might be helpful to put the sum over x 's on the outer sum and the sum over y 's on the inner sum.]

6. Let $Y = \lfloor X \rfloor$ denote the largest integer that is less than or equal to X . For instance: $\lfloor 7.2 \rfloor = 7$, and $\lfloor 2.99 \rfloor = 2$ and $\lfloor 4 \rfloor = 4$. Now suppose that X is an Exponential random variable with $\mathbb{E}(X) = 1/3$.
 - 6a. Find $P(Y \geq 1)$.
 - 6b. Find $P(Y \geq 5)$.
 - 6c. Find $P(Y \geq 10)$.
 - 6d. Can you generalize? What is $P(Y \geq x)$, when x is a (nonnegative) integer?
 - 6e. What kind of discrete random variable is Y ?