

STAT/MA 41600
In-Class Problem Set #34: November 7, 2014

- 1.** Consider a Beta random variable X with parameters $\alpha = 3$ and $\beta = 10$.
 - 1a.** Find the CDF $F_X(x)$ of X .
 - 1b.** What is the probability that X is less than $1/2$?

- 2.** Same setup as #1.
 - 2a.** Verify that the probability density function of X is a valid density. Of course $f_X(x) \geq 0$ in this case, so we only need to verify that $\int_0^1 f_X(x) dx = 1$.
 - 2b.** We know $\mathbb{E}(X) = \alpha/(\alpha + \beta)$, but it is “good practice” to derive it in specific cases, without using this formula. Please derive $\mathbb{E}(X)$, in this case, i.e., with $\alpha = 3$ and $\beta = 10$.

- 3.** Consider a Beta random variable X with parameters $\alpha = 2$ and $\beta = 2$.
 - 3a.** Find the probability that X exceeds $1/3$ (i.e., is larger than $1/3$).
 - 3b.** Find $P(1/4 < X < 3/4)$.

- 4.** Same setup as #3.
 - 4a.** We know $\mathbb{E}(X) = \alpha/(\alpha + \beta)$, but it is “good practice” to derive it in specific cases, without using this formula. Please derive $\mathbb{E}(X)$, in this case, i.e., with $\alpha = 2$ and $\beta = 2$.
 - 4b.** Calculate $\mathbb{E}(X^2)$.
 - 4c.** Use your solutions to **4a** and **4b** to derive $\text{Var}(X)$.

- 5.** Review question: Suppose that U is a (continuous) Uniform random variable on the interval $[0, 1]$, and X is an Exponential random variable with $\mathbb{E}(X) = 1/3$. Also suppose that U and X are independent. Find $P(X > U)$.

- 6.** Review question: Suppose that X_1 and X_2 are independent Exponential random variables with $\mathbb{E}(X_1) = 1/\lambda_1$ and $\mathbb{E}(X_2) = 1/\lambda_2$. Find $P(X_1 > X_2)$.