

STAT/MA 41600
In-Class Problem Set #35: November 10, 2014

1. Consider a standard Normal random variable Z .
 - 1a. Find $P(-1.5 < Z < 1.5)$.
 - 1b. Find $P(-2.5 < Z < 2.5)$.

2. Suppose that X is a Normal random variable with $\mathbb{E}(X) = 3.6$ and $\sigma_X = 2.2$.
 - 2a. Find the probability that X is positive.
 - 2b. Find the probability that the value of X is contained in the interval $[3, 4]$.

3. Consider a Normal random variable X with $\mathbb{E}(X) = 5.7$ and $\text{Var}(X) = 4$.
 - 3a. Find the value of a so that $P(X < a) = 0.05$.
 - 3b. Find the value of b so that $P(X > b) = 0.05$.
[Thus, combining **3a** and **3b**, we have $P(a < X < b) = 0.90$.]

4. Suppose that X is a Normal random variable with $\mathbb{E}(X) = 3$ and $\text{Var}(X) = 4$.
Define $Y = 5X + 1$.
 - 4a. Find $P(10 < Y < 20)$.
 - 4b. Find the probability that Y is negative.

5. Suppose that the heights of blades of grass are Normally distributed and independent, with each height having expected value 4 inches and standard deviation 0.75 inches. A child picks the blades of grass.
 - 5a. What is the probability that the height of 1 blade of grass is 5 inches or taller?
 - 5b. How many blades of grass does the child expect to pick until the first blade of grass appears with height 5 inches or taller?
 - 5c. Now, instead of the setup in **5b**, suppose that the child picks exactly 10 blades of grass. What is the expected number of blades of grass that have height 5 inches or taller, within this collection of 10 blades of grass?

6. Assume that the heights of college females are Normally distributed, with expected height of 64 inches and standard deviation of 4.8 inches.
 - 6a. What is the probability that a college female is 67 inches tall or taller?
 - 6b. If 40 college females are present in a classroom, and X denotes the number of those females who are 67 inches tall or taller, find $\text{Var}(X)$.
 - 6c. What is the probability that the height of a college female is 60 inches or shorter?
 - 6d. Suppose that Y denotes the number of college females we would need to interview, until we find one who is 60 inches or shorter. What is $\text{Var}(Y)$?