

STAT/MA 41600
In-Class Problem Set #35: November 10, 2014
Solutions by Mark Daniel Ward

1a. We have $P(-1.5 < Z < 1.5) = P(Z < 1.5) - P(Z < -1.5)$. From the Normal distribution table, we have $P(Z < 1.5) = 0.9332$ and $P(Z < -1.5) = P(Z > 1.5) = 1 - P(Z < 1.5) = 1 - 0.9332 = 0.0668$. Thus $P(-1.5 < Z < 1.5) = 0.9332 - 0.0668 = 0.8664$.

1b. We have $P(-2.5 < Z < 2.5) = P(Z < 2.5) - P(Z < -2.5)$. From the table, we know $P(Z < 2.5) = 0.9938$ and $P(Z < -2.5) = P(Z > 2.5) = 1 - P(Z < 2.5) = 1 - 0.9938 = 0.0062$. Thus $P(-2.5 < Z < 2.5) = 0.9938 - 0.0062 = 0.9876$.

2a. We have $P(X > 0) = P\left(\frac{X-3.6}{2.2} > \frac{0-3.6}{2.2}\right) = P(Z > -1.63) = P(Z < 1.63) = 0.9484$.

2b. We have $P(3 < X < 4) = P\left(\frac{3-3.6}{2.2} < \frac{X-3.6}{2.2} < \frac{4-3.6}{2.2}\right) = P(-0.27 < Z < 0.18) = P(Z < 0.18) - P(Z < -0.27)$. We know $P(Z < 0.18) = 0.5714$ and $P(Z < -0.27) = P(Z > 0.27) = 1 - P(Z < 0.27) = 1 - 0.6064 = 0.3936$. Thus $P(3 < X < 4) = 0.5714 - 0.3936 = 0.1778$.

3a. We have $0.05 = P(X < a) = P\left(\frac{X-5.7}{2} < \frac{a-5.7}{2}\right) = P(Z < \frac{a-5.7}{2}) = P(Z > -\frac{a-5.7}{2})$. Thus $0.95 = P(Z < -\frac{a-5.7}{2})$, so $-\frac{a-5.7}{2} = 1.65$, so $a = 2.4$.

3b. We have $0.05 = P(X > b) = P\left(\frac{X-5.7}{2} > \frac{b-5.7}{2}\right) = P(Z > \frac{b-5.7}{2})$. Thus $0.95 = P(Z < \frac{b-5.7}{2})$, so $\frac{b-5.7}{2} = 1.65$, so $b = 9$.

4a. Since X is Normal, then $Y = 5X + 1$ is Normal too, with $\mathbb{E}(Y) = \mathbb{E}(5X + 1) = 5\mathbb{E}(X) + 1 = 5(3) + 1 = 16$ and $\text{Var}(Y) = \text{Var}(5X + 1) = 5^2 \text{Var}(X) = 25(4) = 100$. Thus $P(10 < Y < 20) = P\left(\frac{10-16}{10} < \frac{Y-16}{10} < \frac{20-16}{10}\right) = P(-0.6 < Z < 0.4) = P(Z < 0.4) - P(Z < -0.6)$. We have $P(Z < 0.4) = 0.6554$ and $P(Z < -0.6) = P(Z > 0.6) = 1 - P(Z < 0.6) = 1 - 0.7257 = 0.2743$. Thus $P(10 < Y < 20) = 0.6554 - 0.2743 = 0.3811$.

4b. We have $P(Y < 10) = P\left(\frac{Y-16}{10} < \frac{0-16}{10}\right) = P(Z < -1.6) = P(Z > 1.6) = 1 - P(Z < 1.6) = 1 - 0.9452 = 0.0548$.

5a. The probability a blade of grass, with height X , is 5 inches or taller, is $P(X > 5) = P\left(\frac{X-4}{0.75} > \frac{5-4}{0.75}\right) = P(Z > 1.33) = 1 - P(Z < 1.33) = 1 - 0.9082 = 0.0918$.

5b. The number of blades of grass picked until the first blade of grass has height 5 inches or taller is Geometric with expected value $1/0.0918 = 10.89$.

5c. The number of blades of grass, with height 5 inches or taller, among the 10 blades of grass, is Binomial with parameters $n = 10$ and $p = 0.0918$, and therefore has expected value $np = (10)(0.0918) = 0.918$.

6a. The probability that a college female with height V is 67 inches tall or taller is $P(V > 67) = P\left(\frac{V-64}{4.8} > \frac{67-64}{4.8}\right) = P(Z > 0.63) = 1 - P(Z < 0.63) = 1 - 0.7357 = 0.2643$.

6b. Since X is Binomial with $n = 40$ and $p = 0.2643$, then $\text{Var}(X) = npq = (40)(0.2643)(1 - 0.2643) = 7.78$.

6c. The probability that a college female with height V is 60 inches or shorter is $P(V < 60) = P\left(\frac{V-64}{4.8} < \frac{60-64}{4.8}\right) = P(Z < -0.83) = P(Z > 0.83) = 1 - P(Z < 0.83) = 1 - 0.7967 = 0.2033$.

6d. Since Y is Geometric with $p = 0.2033$, then $\text{Var}(Y) = q/p^2 = (1 - 0.2033)/(0.2033)^2 = 19.28$.