STAT/MA 41600
In-Class Problem Set #36: November 12, 2014

1. Suppose that the heights of blades of grass are Normally distributed and independent, with each height having expected value 4 inches and standard deviation 0.75 inches. A child picks the blades of grass. Let X_j denote the height of the jth piece.
1a. If she picks ten blades of grass, let $Y = X_1 + \ldots + X_{10}$ denote the sum of their heights. Find $P(Y < 42.5)$.
1b. Find $P(39.5 < Y < 40.5)$.

2. Assume that the heights of college females are Normally distributed, with expected height of 64 inches and standard deviation of 4.8 inches.
2a. If the heights of 100 students are measured, and X denotes the sum of these 100 heights, find $P(6350 < X < 6450)$.
2b. Find $P(6390 < X < 6410)$.

3. Suppose 40 students in a course have grades that are independent and Normally distributed, each with mean 70 and variance 81. The professor sums the scores across the 40 students. Let Y denote this sum of the 40 grades.
3a. Find $P(2750 < Y < 2850)$.
3b. Find the probability that Y is within one standard deviation of its mean, i.e., find $P(|Y - \mathbb{E}(Y)| < \sigma_Y) = P(\mathbb{E}(Y) - \sigma_Y < Y < \mathbb{E}(Y) + \sigma_Y)$.

4. Suppose that X and Y represent the grades of two students, and X and Y are independent, Normal random variables. Suppose X has $\mathbb{E}(X) = 75$ and $\text{Var}(X) = 50$, and Y has $\mathbb{E}(Y) = 70$ and $\text{Var}(Y) = 60$.
4a. Find $P(Y < X < Y + 3)$.
4b. Is it possible that $X < Y$, even though $\mathbb{E}(X) > \mathbb{E}(Y)$? Calculate $P(X < Y)$.

5. Suppose Juanita wants to buy 10 toy cars, with weights that are independent and Normally distributed, each with expected value 35 grams and standard deviation of 12 grams.

Also suppose that Hector wants to buy a puzzle, and the weight of the puzzle is Normally distributed, with expected value 400 grams and standard deviation of 75 grams.
5a. What is the probability that sum of the weights of the 10 toy cars is heavier (i.e., weighs more) than the puzzle?
5b. What is the probability that the sum of the weights of the 10 toy cars is less than 325 grams?

6. Suppose $X_1, \ldots, X_{200}$ are independent, Normally distributed random variables with $\mathbb{E}(X_j) = 3$ and $\sigma_X^2 = 25$.
6a. Find $P(X_1 + \cdots + X_{200} < 700)$.
6b. Let $Y = (X_1 + \cdots + X_{200})/200$ be the average of the X_j's. Find $P(|Y - 3| \leq 0.2)$.