STAT/MA 41600
In-Class Problem Set #37: November 14, 2014

1. Suppose that $X_1, \ldots, X_{300}$ are independent Exponential random variables, each having $\mathbb{E}(X_j) = 1/5$.
1a. What kind of random variable is $X_1 + \cdots + X_{300}$?
1b. What are the expected value, variance, and standard deviation of $X_1 + \cdots + X_{300}$?
1c. Find a good approximation for $P(58 < X_1 + \cdots + X_{300} < 62)$.
1d. Suppose that Y is a Normal random variable, independent from the X_j's, with expected value 63 and variance 10. Calculate a good estimate for $P(Y < X_1 + \cdots + X_{300})$.

2. Assume that X and Y are two independent Gamma random variables, where X has parameters $\lambda_X = 2$ and $r_X = 200$, and where Y has parameters $\lambda_Y = 3$ and $r_Y = 312$. (We just put some subscripts on the parameters, to try to help you tell which parameters go with which random variables here.) Find a good approximation for $P(X < Y)$.

3. Suppose that $U_1, \ldots, U_{50}$ are independent, continuous random variables, each of which is Uniformly distributed on the interval $[0, 6]$.
3a. Find a good approximation for $P(140 < U_1 + \cdots + U_{50} < 160)$.
3b. Find a good approximation for $P(|U_1 + \cdots + U_{50} - 150| < 5)$.

4. Suppose that $X_1, \ldots, X_{500}$ are independent Geometric random variables, each of which have $\mathbb{E}(X_j) = 8/5$.
4a. What kind of random variable is $X_1 + \cdots + X_{500}$?
4b. What are the expected value, variance, and standard deviation of $X_1 + \cdots + X_{500}$?
4c. Find a good approximation for $P(780 < X_1 + \cdots + X_{500} < 820)$.
4d. Suppose that Y is a Negative Binomial random variable, independent from the X_j's, with parameters $r = 250$ and $p = 1/3$. Calculate a good estimate for $P(Y < X_1 + \cdots + X_{500})$.

5. If X is a Poisson random variable with $\lambda = 300$, estimate $P(290 < X < 310)$.

6. Define $f(x) = x^2/72$ for $0 < x < 6$ and $f(x) = 0$ otherwise. Suppose $X_1, \ldots, X_{100}$ are independent, continuous random variables that each have the density $f(x)$.
6a. Find $\mathbb{E}(X_j)$.
6b. Find $\mathrm{Var}(X_j)$.
6c. Find a good estimate for $P(X_1 + \cdots + X_{100} < 460)$.