STAT/MA 41600
In-Class Problem Set #37 part 2: November 17, 2014

1. If X and Y are two independent Gamma random variables that each have parameters $r = 500$ and $\lambda = 5$, find the probability that X and Y differ by 3 or more. In other words, find $P(|X - Y| \geq 3)$.

2. If X and Y are two independent Negative Binomial random variables that each have parameters $r = 100$ and $p = 4/5$, find the probability that X and Y differ by 3 or more. In other words, find $P(|X - Y| \geq 3)$.

3. If X and Y are two independent Poisson random variables that each have parameters $\lambda = 800$, find the probability that X and Y differ by 25 or more. In other words, find $P(|X - Y| \geq 25)$.

4. Suppose that X is a Binomial random variable with $n = 20{,}000$ and $p = 1/2500$.
4a. Find a Poisson approximation for $P(X \geq 5)$.
4b. Find a Normal approximation for $P(X \geq 5)$.

5. During a 365-day year, Alain looks for rain every day. Each day, Alain estimates that there is rain with probability 1/4, and that the chances on separate days are independent. Similarly, in a different city, Brent looks for rain every day. His chances of rain are also assumed to be independent from day to day, and are independent from Alain's chances; he has an estimated chance of 1/5 rain on a given day. Let X and Y denote the number of rainy days seen by Alain and Brent during the year, respectively.
5a. Write an exact expression of $P(X > Y)$. You do not need to evaluate the expression.
5b. Find a good estimate for $P(X > Y)$.

6. Assume that $U_1, \ldots, U_{800}$ are independent continuous random variables that are each Uniformly distributed on $[0, 6]$. Also assume that X is a Normal random variable, independent from the U_j's, with $\mathbb{E}(X) = 2500$ and $\mathrm{Var}\,(X) = 900$. Find a good estimate for $P(X > U_1 + \cdots + U_{800})$.