STAT/MA 41600
In-Class Problem Set #37 part 2: November 17, 2014
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1. We split into two nonoverlapping events: $P(|X-Y| \geq 3) = P(X-Y \geq 3) + P(Y-X \geq 3)$. We can make a good approximation $P(X-Y \geq 3) = P(\frac{X-Y-(100-100)}{\sqrt{20+20}} \geq \frac{3-(100-100)}{\sqrt{20+20}}) \approx P(Z \geq 0.47) = 1 - P(Z \leq 0.47) = 1 - 0.6808 = 0.3192$. Thus $P(|X-Y| \geq 3) = (2)(0.3192) = 0.6384$.

2. We split into two nonoverlapping events: $P(|X-Y| \geq 3) = P(X-Y \geq 3) + P(Y-X \geq 3)$. Since we are using a continuous random variable as an approximation for a discrete random variable, we use continuity correction. We can make a good approximation $P(X-Y \geq 3) = P(X-Y \geq 2.5) = P(\frac{X-Y-(125-125)}{\sqrt{125/4+125/4}} \geq \frac{2.5-(125-125)}{\sqrt{125/4+125/4}}) \approx P(Z \geq 0.32) = 1 - P(Z \leq 0.32) = 1 - 0.6255 = 0.3745$. Thus $P(|X-Y| \geq 3) = (2)(0.3745) = 0.7490$.

3. We split into two nonoverlapping events: $P(|X-Y| \geq 25) = P(X-Y \geq 25) + P(Y-X \geq 25)$. Since we are using a continuous random variable as an approximation for a discrete random variable, we use continuity correction. We can make a good approximation $P(X-Y \geq 25) = P(X-Y \geq 24.5) = P(\frac{X-Y-(800-800)}{\sqrt{800+800}} \geq \frac{24.5-(800-800)}{\sqrt{800+800}}) \approx P(Z \geq 0.61) = 1 - P(Z \leq 0.61) = 1 - 0.7291 = 0.2709$. Thus $P(|X-Y| \geq 25) = (2)(0.2709) = 0.5418$.

4a. Since n is large, p is small, and npq has a moderate size, then we define Y to be a Poisson random variable with parameter $\lambda = np = 8$, and we have $P(X \geq 5) \approx P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - \sum_{y=0}^{4} e^{-8} 8^y / y! = 0.9004$.
4b. Since we are using a continuous random variable as an approximation for a discrete random variable, we use continuity correction. We have $P(X \geq 5) = P(X \geq 4.5) = P(\frac{X-8}{\sqrt{7.9968}} \geq \frac{4.5-8}{\sqrt{7.9968}}) \approx P(Z \geq \frac{4.5-8}{\sqrt{8}}) = P(Z \geq -1.24) = P(Z \leq 1.24) = 0.8925$.

5a. An exact expression is $P(X > Y) = \sum_{x=1}^{365} \sum_{y=0}^{x-1} \binom{365}{x} (1/4)^x (3/4)^{365-x} (1/5)^y (4/5)^{365-y}$.
5b. An approximation—*using continuity correction*, since we are approximating a discrete random variable with a continuous random variable—is $P(X > Y) = P(X - Y > 0) = P(X - Y > 0.5) = P(\frac{X-Y-((365)(1/4)-(365)(1/5))}{\sqrt{(365)(1/4)(3/4)+(365)(1/5)(4/5)}} > \frac{0.5-((365)(1/4)-(365)(1/5))}{\sqrt{(365)(1/4)(3/4)+(365)(1/5)(4/5)}}) \approx P(Z > -1.57) = P(Z < 1.57) = 0.9418$.

6. We estimate $P(X > U_1 + \cdots + U_{800}) = P(X - (U_1 + \cdots + U_{800}) > 0) = P(\frac{X-(U_1+\cdots+U_{800})-(2500-800(3))}{\sqrt{900+800(3)}} > \frac{0-(2500-800(3))}{\sqrt{900+800(3)}}) \approx P(Z > -1.74) = P(Z < 1.74) = 0.9591$.