STAT/MA 41600
In-Class Problem Set #39: November 24, 2014

1. Roll two 6-sided dice. Let X denote the minimum value, and let Y denote the maximum value.
1a. Find the covariance of X and Y.
1b. Find the correlation of X and Y.

2. Suppose that 6 students come to a party, and each of them brings their favorite CD with them. (For convenience, assume that they each have a different favorite CD.) As they leave the party, they take a CD at random as they depart. Let X denote the number of students who get their own CD back. Find $\mathrm{Var}(X)$.

3. Draw 5 cards from a 52 card deck (without replacement).
3a. Let X denote the number of Hearts that appear. Find $\mathbb{E}(X)$ and also $\mathrm{Var}(X)$.
3b. Let Y denote the number of Queens that appear. Find $\mathbb{E}(Y)$ and also $\mathrm{Var}(Y)$.
3c. Find the covariance of X and Y.

4. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices located at the points $(0,0), (3,0), (5,2), (0,2)$.
4a. Find the covariance of X and Y.
4b. Find the correlation of X and Y.

5. Suppose that X and Y have a constant joint density on the triangle with vertices $(0,0), (3,0), (0,3)$.
5a. Find the covariance of X and Y.
5b. Find the correlation of X and Y.

6. Suppose X and Y have joint probability density function

$$f_{X,Y}(x,y) = 60e^{-4x-6y}$$

for $0 < x < y$; and $f_{X,Y}(x,y) = 0$ otherwise. We already saw, on Midterm Exam #2, that $\mathbb{E}(X) = 1/10$.
6a. Find $\mathbb{E}(Y)$.
6b. Find $\mathbb{E}(XY)$.
6c. Use what you know to find the covariance of X and Y.