

STAT/MA 41600
In-Class Problem Set #40: December 1, 2014

1. Roll two 6-sided dice. Let X denote the maximum value, and let Y denote the minimum value.

1a. Find $\mathbb{E}(X \mid Y = 5)$.

1b. Find $\mathbb{E}(Y \mid X = 5)$.

2. Suppose that X and Y have joint probability mass function

$$p_{X,Y}(x, y) = (2/3)^x(1/2)^y \quad \text{for integers } 1 \leq x \leq y,$$

and $p_{X,Y}(x, y) = 0$ otherwise.

2a. Find the probability mass function of X .

2b. Find the conditional probability mass function $p_{Y|X}(y \mid x)$ of Y , given $X = x$.

2c. Check that $p_{Y|X}(y \mid x)$ is a valid probability mass function, i.e., that it is nonnegative and sums to 1, i.e., check that $\sum_{y=x}^{\infty} p_{Y|X}(y \mid x) = 1$.

2d. Find $\mathbb{E}(Y \mid X = x)$, for a fixed $x \geq 1$.

3. Suppose that we draw a random number Y of cards from a standard deck of cards (with replacement and shuffling after each draw), and suppose that $\mathbb{E}(Y) = 10$. Let X denote the number of Hearts that appear altogether during these Y draws from the deck.

3a. Find $\mathbb{E}(X \mid Y = y)$.

3b. Find $\mathbb{E}(X)$.

4. Consider a pair of random variables X, Y with constant joint density on the quadrilateral with vertices located at the points $(0, 0), (3, 0), (5, 2), (0, 2)$.

4a. Find the probability density function of Y .

4b. Find the conditional probability density function $f_{X|Y}(x \mid y)$ of X , given $Y = y$.

4c. Check that $f_{X|Y}(x \mid y)$ is a valid probability density function, i.e., that it is nonnegative and integrates to 1, i.e., check that $\int_0^{y+3} f_{X|Y}(x \mid y) dx = 1$.

4d. Find $\mathbb{E}(X \mid Y = y)$, for a fixed y with $0 \leq y \leq 2$.

5. Suppose that we draw two cards from a standard deck, without replacement. Let X indicate whether the first card is a Queen, i.e., $X = 1$ if the first card is a Queen, and $X = 0$ otherwise. Similarly, let Y indicate if the second card is a Queen, i.e., $Y = 1$ if the second card is a Queen, and $Y = 0$ otherwise.

5a. Find $\mathbb{E}(Y \mid X = 1)$.

5b. Find $\mathbb{E}(Y \mid X = 0)$.

5c. Find $\mathbb{E}(X \mid Y = 1)$.

5d. Find $\mathbb{E}(X \mid Y = 0)$.

6. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 60e^{-4x-6y}$$

for $0 < x < y$; and $f_{X,Y}(x, y) = 0$ otherwise.

6a. Find $\mathbb{E}(Y \mid X = x)$, for a fixed $x > 0$.

6a. Find $\mathbb{E}(X \mid Y = y)$, for a fixed $y > 0$.