

STAT/MA 41600  
In-Class Problem Set #40: December 1, 2014

1. Roll two 6-sided dice. Let  $X$  denote the maximum value, and let  $Y$  denote the minimum value.

1a. Find  $\mathbb{E}(X \mid Y = 5)$ .

1b. Find  $\mathbb{E}(Y \mid X = 5)$ .

2. Suppose that  $X$  and  $Y$  have joint probability mass function

$$p_{X,Y}(x, y) = (2/3)^x(1/2)^y \quad \text{for integers } 1 \leq x \leq y,$$

and  $p_{X,Y}(x, y) = 0$  otherwise.

2a. Find the probability mass function of  $X$ .

2b. Find the conditional probability mass function  $p_{Y|X}(y \mid x)$  of  $Y$ , given  $X = x$ .

2c. Check that  $p_{Y|X}(y \mid x)$  is a valid probability mass function, i.e., that it is nonnegative and sums to 1, i.e., check that  $\sum_{y=x}^{\infty} p_{Y|X}(y \mid x) = 1$ .

2d. Find  $\mathbb{E}(Y \mid X = x)$ , for a fixed  $x \geq 1$ .

3. Suppose that we draw a random number  $Y$  of cards from a standard deck of cards (with replacement and shuffling after each draw), and suppose that  $\mathbb{E}(Y) = 10$ . Let  $X$  denote the number of Hearts that appear altogether during these  $Y$  draws from the deck.

3a. Find  $\mathbb{E}(X \mid Y = y)$ .

3b. Find  $\mathbb{E}(X)$ .

4. Consider a pair of random variables  $X, Y$  with constant joint density on the quadrilateral with vertices located at the points  $(0, 0), (3, 0), (5, 2), (0, 2)$ .

4a. Find the probability density function of  $Y$ .

4b. Find the conditional probability density function  $f_{X|Y}(x \mid y)$  of  $X$ , given  $Y = y$ .

4c. Check that  $f_{X|Y}(x \mid y)$  is a valid probability density function, i.e., that it is nonnegative and integrates to 1, i.e., check that  $\int_0^{y+3} f_{X|Y}(x \mid y) dx = 1$ .

4d. Find  $\mathbb{E}(X \mid Y = y)$ , for a fixed  $y$  with  $0 \leq y \leq 2$ .

5. Suppose that we draw two cards from a standard deck, without replacement. Let  $X$  indicate whether the first card is a Queen, i.e.,  $X = 1$  if the first card is a Queen, and  $X = 0$  otherwise. Similarly, let  $Y$  indicate if the second card is a Queen, i.e.,  $Y = 1$  if the second card is a Queen, and  $Y = 0$  otherwise.

5a. Find  $\mathbb{E}(Y \mid X = 1)$ .

5b. Find  $\mathbb{E}(Y \mid X = 0)$ .

5c. Find  $\mathbb{E}(X \mid Y = 1)$ .

5d. Find  $\mathbb{E}(X \mid Y = 0)$ .

6. Suppose  $X$  and  $Y$  have joint probability density function

$$f_{X,Y}(x, y) = 60e^{-4x-6y}$$

for  $0 < x < y$ ; and  $f_{X,Y}(x, y) = 0$  otherwise.

6a. Find  $\mathbb{E}(Y \mid X = x)$ , for a fixed  $x > 0$ .

6a. Find  $\mathbb{E}(X \mid Y = y)$ , for a fixed  $y > 0$ .