STAT/MA 41600
In-Class Problem Set #41: December 3, 2014

1. Suppose that the waiting time for the bus is, on average, 8 minutes.
1a. Find an upper bound for the probability that a student spends 10 minutes (or more) waiting for the bus.
1b. If we also know that the waiting time has standard deviation 1.5 minutes, find a lower-bound on the probability that the waiting time is between 6.5 minutes and 9.5 minutes.
1c. (Simple curiosity question.) Is it possible that the waiting time is an Exponential random variable? Why or why not? (The only things we know about the waiting time is that it has average of 8 minutes and standard deviation of 1.5 minutes.)

2. Juanita carefully studies the distribution of the time between consecutive emails arriving. Suppose that the average time between consecutive emails has expected value 2.5 minutes, with a standard deviation of 1.5 minutes. Find a bound on the probability that the waiting time between two of her emails is 2 to 3 minutes.

3. Suppose that a randomly chosen student sleeps, on average, 6 hours per night. Also suppose that the standard deviation of the time that a student sleeps on a given night is 1.3 hours. If a student gets between 5 to 7 hours of sleep in a night, they are happy. Find a bound on the probability that a student is unhappy after a night's sleep (because they got either too much or too little sleep!).

4. (Review question) Suppose that $X_1, \ldots, X_{250}$ are independent geometric random variables, with $\mathbb{E}(X_j) = 3$ for each j with $1 \leq j \leq 250$.
4a. Find an exact expression for $P(730 \leq X_1 + \cdots + X_{250} \leq 770)$. (Do not evaluate it!)
4b. Approximate the value of $P(730 \leq X_1 + \cdots + X_{250} \leq 770)$. (Please, do give a value.)

5. (Review question) Suppose that the joint probability density function of X and Y is constant on the triangle with corners at $(0,0)$, $(3,0)$, and $(0,6)$.
5a. Find $P(Y < 1 \mid X < 1)$.
5b. Find $P(Y < 1 \mid X = 1)$.
5c. Find $\mathbb{E}(Y \mid X = 1)$.

6. (Review question) Suppose that two students named Gwyneth and Josephine have a total of 20 CDs in their room, consisting of 5 blues CDs and 15 reggae CDs. Each of the students chooses 7 CDs at random (without replacement), with all choices equally likely. (Thus, altogether, 6 CDs remain unchosen.) Let X denote the number of blues CDs that Gwyneth selects. Let Y denote the number of blues CDs that Josephine selects. Find the covariance between X and Y.