

STAT/MA 41600  
In-Class Problem Set #42: December 5, 2014

- 1.** Consider three independent Exponential random variables  $X_1, X_2, X_3$ , each with mean 1.
  - 1a.** Find the density of  $X_{(1)} = \min(X_1, X_2, X_3)$ .
  - 1b.** Compute  $\mathbb{E}(X_{(1)})$ .
  - 1c.** Find the density of the second order statistic,  $X_{(2)}$ , i.e., the second-smallest one.
  - 1d.** Compute  $\mathbb{E}(X_{(2)})$ .
  
- 2.** Same setup as in **1**.
  - 2a.** Find the density of  $X_{(3)} = \max(X_1, X_2, X_3)$ .
  - 2b.** Compute  $\mathbb{E}(X_{(3)})$ .
  - 2c.** Sanity check: We know that  $X_1 + X_2 + X_3 = X_{(1)} + X_{(2)} + X_{(3)}$ . Therefore, we have  $\mathbb{E}(X_{(1)}) + \mathbb{E}(X_{(2)}) + \mathbb{E}(X_{(3)}) = \mathbb{E}(X_1 + X_2 + X_3) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) = 1 + 1 + 1 = 3$ . So please make sure your answers to **1b**, **1d**, and **2b** sum to 3 too.
  
- 3.** Consider a circle of radius 3. Let  $X_1$  and  $X_2$  be two points, each chosen Uniformly at random in the circle. Let  $W_1$  and  $W_2$  be their respective distances to the origin.
  - 3a.** Find the CDF of  $W_1$ . (By symmetry,  $W_2$  has the same CDF.)
  - 3b.** Find the density of  $W_1$ . (By symmetry,  $W_2$  has the same density too.)
  - 3c.** Find the expected value of  $W_1$ . (By symmetry,  $W_2$  has the same expected value too.)
  
- 4.** Same setup as in **3**. Let  $W_{(1)}$  and  $W_{(2)}$  be the order statistics of the pair  $W_1, W_2$ .
  - 4a.** Find the density of  $W_{(1)} = \min(W_1, W_2)$ .
  - 4b.** Compute  $\mathbb{E}(W_{(1)})$ .
  - 4c.** Find the density of  $W_{(2)} = \max(W_1, W_2)$ .
  - 4d.** Compute  $\mathbb{E}(W_{(2)})$ .
  - 4e.** Sanity check: We know that  $W_1 + W_2 = W_{(1)} + W_{(2)}$ . So  $\mathbb{E}(W_{(1)}) + \mathbb{E}(W_{(2)}) = \mathbb{E}(W_1 + W_2) = \mathbb{E}(W_1) + \mathbb{E}(W_2)$ . So please make sure that your answers to **4b** and **4d** have the same sum as we would find if we compared to  $\mathbb{E}(W_1) + \mathbb{E}(W_2)$  from **3c**.
  
- 5.** Let  $U_1, U_2, U_3, U_4, U_5$  be five independent, continuous random variables, each uniformly distributed on  $[0, 10]$ . Then  $U_{(4)}$  denotes the second-largest of these five random variables.
  - 5a.** Find the probability density function of  $U_{(4)}$ .
  - 5b.** Find the mean of  $U_{(4)}$ .
  
- 6.** (Review question) Suppose that heights of blades of grass have expected value 4 inches and standard deviation 0.75 inches. (Do not assume that the heights are Normally distributed).
  - 6a.** Find a bound on the probability that a randomly selected blade of grass is at least 6.5 inches tall.
  - 6b.** A recent commercial says that this kind of grass has a *good looking appearance* when it is between 2.75 to 5.25 inches tall. Find a bound on the probability that a randomly selected blade of grass has this type of *good looking appearance*.