1. Consider three independent Exponential random variables $X_1, X_2, X_3$, each with mean 1.

1a. Find the density of $X_{(1)} = \min(X_1, X_2, X_3)$.

1b. Compute $E(X_{(1)})$.

1c. Find the density of the second order statistic, $X_{(2)}$, i.e., the second-smallest one.

1d. Compute $E(X_{(2)})$.

2. Same setup as in 1.

2a. Find the density of $X_{(3)} = \max(X_1, X_2, X_3)$.

2b. Compute $E(X_{(3)})$.

2c. Sanity check: We know that $X_1 + X_2 + X_3 = X_{(1)} + X_{(2)} + X_{(3)}$. Therefore, we have $E(X_{(1)}) + E(X_{(2)}) + E(X_{(3)}) = E(X_1 + X_2 + X_3) = E(X_1) + E(X_2) + E(X_3) = 1 + 1 + 1 = 3$. So please make sure your answers to 1b, 1d, and 2b sum to 3 too.

3. Consider a circle of radius 3. Let $X_1$ and $X_2$ be two points, each chosen Uniformly at random in the circle. Let $W_1$ and $W_2$ be their respective distances to the origin.

3a. Find the CDF of $W_1$. (By symmetry, $W_2$ has the same CDF.)

3b. Find the density of $W_1$. (By symmetry, $W_2$ has the same density too.)

3c. Find the expected value of $W_1$. (By symmetry, $W_2$ has the same expected value too.)

4. Same setup as in 3. Let $W_{(1)}$ and $W_{(2)}$ be the order statistics of the pair $W_1, W_2$.

4a. Find the density of $W_{(1)} = \min(W_1, W_2)$.

4b. Compute $E(W_{(1)})$.

4c. Find the density of $W_{(2)} = \max(W_1, W_2)$.

4d. Compute $E(W_{(2)})$.

4e. Sanity check: We know that $W_1 + W_2 = W_{(1)} + W_{(2)}$. So $E(W_{(1)}) + E(W_{(2)}) = E(W_1 + W_2) = E(W_1) + E(W_2)$. So please make sure that your answers to 4b and 4d have the same sum as we would find if we compared to $E(W_1) + E(W_2)$ from 3c.

5. Let $U_1, U_2, U_3, U_4, U_5$ be five independent, continuous random variables, each uniformly distributed on $[0, 10]$. Then $U_{(4)}$ denotes the second-largest of these five random variables.

5a. Find the probability density function of $U_{(4)}$.

5b. Find the mean of $U_{(4)}$.

6. (Review question) Suppose that heights of blades of grass have expected value 4 inches and standard deviation 0.75 inches. (Do not assume that the heights are Normally distributed.)

6a. Find a bound on the probability that a randomly selected blade of grass is at least 6.5 inches tall.

6b. A recent commercial says that this kind of grass has a good looking appearance when it is between 2.75 to 5.25 inches tall. Find a bound on the probability that a randomly selected blade of grass has this type of good looking appearance.