STAT/MA 41600
In-Class Problem Set #43: December 8, 2014

1. Suppose that X is an Exponential random variable with $\mathbb{E}(X) = 1/3$. Find the moment generating function $g(t) = M_X(t) = \mathbb{E}(e^{tX})$ of X. (It is OK to assume $t < 3$.)

Hint: The thing to compute is $g(t) = M_X(t) = \mathbb{E}(e^{tX}) = \int_0^\infty (e^{tx})(3e^{-3x})\,dx$.

2a. For the moment generating function $g(t) = M_X(t)$ in question **1**, please compute $g'(t) = M_X'(t)$, i.e., compute the derivative of the moment generating function with respect to t.

2b. Compute $g'(0) = M_X'(0)$. Hint: Since this should be equal to $\mathbb{E}(X)$, we should get 1/3 for the solution.

3. Suppose that X has probability density function $f_X(x) = 25xe^{-5x}$ for $x > 0$, and $f_X(x) = 0$ otherwise. Find the moment generating function of X.

4. Suppose that X has moment generating function $g(t) = M_X(t) = \mathbb{E}(e^{tX}) = \frac{25}{t^2-10t+25}$. Find $\mathbb{E}(X)$.

5. If X is a Geometric random variable with probability of success $p = 1/5$ on each trial, find the moment generating function $g(t) = M_X(t)$ of X.

6a. For the moment generating function $g(t) = M_X(t)$ in question **5**, please compute $g'(t) = M_X'(t)$, i.e., compute the derivative of the moment generating function with respect to t.

6b. Compute $g'(0) = M_X'(0)$. Hint: Since this should be equal to $\mathbb{E}(X)$, we should get 5 for the solution.