STAT/MA 41600

Midterm Exam #2: November 21, 2014

Name ____

Purdue student ID (10 digits) ____

1. The testing booklet contains 5 questions, which are all weighted evenly (i.e., each question is worth 1/5 of the midterm exam grade).

2. Permitted Texas Instruments calculators:
   BA-35
   BA II Plus*
   BA II Plus Professional Edition*
   TI-30XS MultiView*
   TI-30Xa
   TI-30XIIS*
   TI-30XIIB*
   TI-30XB MultiView*
   *The memory of the calculator should be cleared at the start of the exam.

3. **Circle your final answer in your booklet**; otherwise, no credit may be given.

4. There is no penalty for guessing or partial work.

5. Show all your work in the exam booklet. If the majority of questions are answered correctly, but insufficient work is given, the exam could be considered for academic misconduct. Therefore, you should *show all your work and justify your solutions* in the exam booklet.

6. Extra sheets of paper are available from the proctor.

You will *not* need these formulas on this particular midterm exam, but nonetheless, Dr Ward promised to mention that a Gamma random variable with parameters λ and r has probability density function:

$$f_X(x) = \begin{cases} \frac{\lambda^r}{(r-1)!} x^{r-1} e^{-\lambda x}, & \text{for } x > 0, \\ 0 & \text{otherwise,} \end{cases}$$

and the cumulative distribution function (CDF) is:

$$F_X(x) = \begin{cases} 1 - e^{-\lambda x} \sum_{j=0}^{r-1} \frac{(\lambda x)^j}{j!}, & \text{for } x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

1. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 12e^{-3x-4y}$$

for $x > 0$ and $y > 0$; and $f_{X,Y}(x, y) = 0$ otherwise. Compute $P(Y > X)$.

2. Suppose X and Y have joint probability density function

$$f_{X,Y}(x, y) = 60e^{-4x-6y}$$

for $0 < x < y$; and $f_{X,Y}(x, y) = 0$ otherwise. (Note that X and Y are *not* independent, since we are insisting that $X < Y$. Since $X < Y$, then X and Y are defined in the portion of the first quadrant that is *above* the line $y = x$.)

Find $\mathbb{E}(X)$.

3. Suppose U and V are independent, continuous random variables, each uniformly distributed on the interval $[0, 10]$. Define $X = \max(U, V)$.

3a. Find the cumulative distribution function of X.

3b. Find the probability density function of X.

3c. Check your own work: Give a brief justification for why your answer to **3b** is a valid probability density function.

4. Suppose that $U_1, \ldots, U_{200}$ are independent, continuous random variables, each of which is Uniformly distributed on the interval $[0, 6]$.

4a. Compute $\mathbb{E}(U_j)$ for a specific, fixed j in the range $1 \leq j \leq 200$.

4b. Compute $\mathbb{E}(U_j^2)$ for a specific, fixed j in the range $1 \leq j \leq 200$.

4c. Use your answers to **4a** and **4b** to compute $\text{Var}\,(U_j)$ for a specific, fixed j in the range $1 \leq j \leq 200$.

4d. Find a good approximation for $P(U_1 + \cdots + U_{200} < 625)$.

5. If X and Y are two independent Poisson random variables that each have parameters $\lambda = 450$, compute a good estimate for $P(Y - X \geq 20)$.