

Different ways to define a sample space. Examples with a series of coin flips.

Flip a coin repeatedly. Might want to, for instance, keep track of all the results on all of the flips. In that case, we would define the sample space

$$S = \{r_1 r_2 r_3 \dots \mid r_j = T \text{ or } r_j = H\}$$

In that case, an example of an outcome is a sequence of  $T$ 's and  $H$ 's. For instance, the outcome might be  $TTTHTHTTTTHH \dots$ . Here, an event such as "the first two flips are  $H$  and then  $T$ , respectively, could be written as

$$A = \{HT r_3 r_4 \dots \mid r_j = T \text{ or } r_j = H\}$$

We could (instead) write the sample space  $S$  in a completely different way, depending on our application. Suppose, for instance, we just really want to know how many flips are needed, to see the first  $H$ . In this case, it makes more sense to write the sample space as

$$S = \{\overbrace{TT \dots T}^{j-1} H \mid j \geq 1\} \cup \{TTTTTTT \dots\}$$

An example of an outcome in this setup looks like, for instance,  $TTTTTTTTTTTH$ . The event that at least 4 flips are needed to see the first head could be written as

$$A = \{\overbrace{TT \dots T}^{j-1} H \mid j \geq 4\} \cup \{TTTTTTT \dots\}$$