

The probability of the empty set is always 0.

Know that  $P(S) = 1$ .

Know that  $S, \emptyset$  (even multiple copies of the empty set) are disjoint. I.e.,

$$S \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \dots = S$$

So we see that

$$P(S \cup \emptyset \cup \emptyset \cup \emptyset \cup \emptyset \cup \dots) = P(S) = 1$$

Since the events are disjoint, we can sum the probability of the individual events to get the probability of the union of the events. I.e.,

$$P(S) + P(\emptyset) + P(\emptyset) + P(\emptyset) + \dots = 1$$

but again  $P(S) = 1$  so

$$1 + P(\emptyset) + P(\emptyset) + P(\emptyset) + \dots = 1$$

So the only possibility is that

$$P(\emptyset) = 0.$$