

The concept of setminus: For example, the notation  $B \setminus A = B \cap A^c$

The concept of [partition](#) of the sample space: We say that  $A_1, A_2, \dots, A_k$  form a partition of the sample space  $S$  if

$$A_1 \cup A_2 \cup \dots \cup A_k = S$$

and (also) the  $A_j$ 's are disjoint.

In particular, if  $k = 2$ , notice that a partition must just have the form  $A \cup A^c = S$ .

The complement of an event  $A$  always has probability  $1 - P(A)$ , i.e.,

$$P(A^c) = 1 - P(A)$$

Why? First write

$$1 = P(S) = P(A \cup A^c)$$

but  $A$  and  $A^c$  are disjoint, so  $P(A \cup A^c) = P(A) + P(A^c)$ . So  $1 = P(A) + P(A^c)$ , and subtract  $P(A)$  on both sides,  $P(A^c) = 1 - P(A)$ .

If we have events  $A$  and  $B$  such that  $A \subset B$ , i.e., such that every outcome of  $A$  is also in  $B$  as well, then  $P(A) \leq P(B)$ . Why? Write  $B = A \cup (B \setminus A)$ . Also  $A$  and  $B \setminus A$  are two disjoint events. So now we have  $B = A \cup (B \setminus A)$  is a disjoint union. So we get

$$P(B) = P(A) + P(B \setminus A) \geq P(A)$$

because  $P(B \setminus A) \geq 0$ , because all probabilities are always 0 or bigger. So we conclude: If  $A \subset B$  then  $P(A) \leq P(B)$ .