

Independence of collections of events

If  $A, B, C$  are events, we define the triplet  $A, B, C$  to be independent if four things (simultaneously) happen:

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cap C) = P(A)P(C)$$

$$P(B \cap C) = P(B)P(C)$$

$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

We also need a method for defining independence for even larger collections of events. If  $A_1, A_2, \dots, A_k$  are events, we say that they are independent if the product of the intersection of any finite subcollection of them is equal to the product of the individual probabilities of them.

If  $A_1, A_2, A_3, \dots$  are an infinite sequence of events, say that they are independent if every finite subcollection of them is independent too.

Remark: If, for instance,  $A$  and  $B$  are independent events, then it is straightforward to check that  $A$  and  $B^c$  are independent, and also  $A^c$  and  $B$  are independent, and also  $A^c$  and  $B^c$  are independent. Similar nice properties extend to largest collections of events as well.