Examples of independent events. Roll a die repeatedly, let the jth event be something about only the jth roll.

Draw cards from a deck of 52 cards altogether, replacing and reshuffling the chosen card each time. If the jth event only concerns the jth draw, then the events are quite likely to be independent.

For example, if we pick shapes from a bag, and the jth event concerns the jth draw, if the shapes are replaced in between draws, then the events are independent.

Flip coins, over and over, if the jth event deals with the jth flip only, then the events should be independent.

In particular, if there is an infinite (but could be finite) sequence of things that we try over and over, and if the probabilities do not change from try to try, we usually call such things trials.

Suppose we roll a 6-sided standard die repeatedly. Suppose we want to know the probability that the first occurrence of a 3 happens sometime before the first occurrence of a 5. (Not necessarily immediately before.) I.e., what is the probability I see a 3 before I ever see a 5? We can classify these trials into 3 types:

"good trial" is one in which 3 appears,

"bad trial" is one in which 5 appears,

and a "neutral trial" is one in which 1, 2, 4, or 6 appears.

In order to have the first 3 appear sometime before the first 5, what we need to have happen is:

For some $n \geq 1$, we need exactly $n-1$ neutral trials, and then a good trial. In other words, we need exactly $n-1$ die rolls that show 1, 2, 4, or 6, followed by a 3 on the nth roll.

All of those events would be disjoint, since the first 3 occurs on the nth roll of each such event. So we can add the probabilities of these events.

So the probability of the first 3 appearing sometime before the first 5 is

$$\sum_{n=1}^{\infty}(4/6)^{n-1}(1/6) = (1/6)\frac{1}{1-4/6} = \frac{1/6}{2/6} = 1/2$$

Intuitively, set aside all of the 1's, 2's, 4's, and 6's, the first time a 3 or 5 appears, it is a 3 with probability 1/2 or a 5 with probability 1/2.